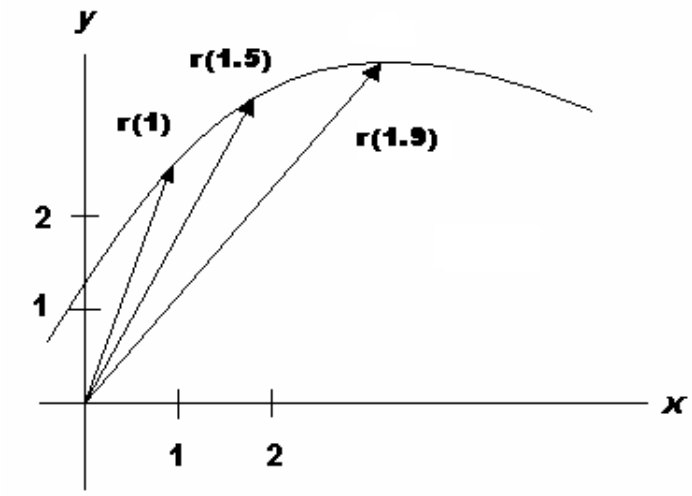
Homework Section 13.4

1. The picture below shows the path of an object. The object's location is given by the position vector function $\mathbf{r}(t)$, where t is time.

   a) Sketch the average velocity vector from time $t = 1$ to $t = 1.5$.
   b) Sketch the average velocity vector from time $t = 1.5$ to $t = 1.9$.
   c) Write an expression for the velocity vector $\mathbf{v}(1.5)$.
   d) Draw an approximation to the vector $\mathbf{v}(1.5)$ and estimate the speed of the particle at $t = 1.5$.



2. Let $\mathbf{r}(t) = 2t\mathbf{i} + (1 - t^2)\mathbf{j}$
   a) Calculate the velocity, acceleration, and speed of an object with position function $\mathbf{r}(t)$.
   b) Sketch the object's path and draw the velocity and acceleration vectors for $t = 2$. Place the initial points of $\mathbf{v}(2)$ and $\mathbf{a}(2)$ on $\mathbf{r}(2)$

3. Calculate the velocity, acceleration, and speed of an object with the given position function.

   a) $\mathbf{r}(t) = \langle 3t^2, 1 + 2t, 4 - t^3 \rangle$ b) $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}$

4. Find the velocity and position functions corresponding to the acceleration function $\mathbf{a}(t) = \mathbf{j}$ with initial conditions $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{j} - \mathbf{k}$.

5. Find out when a particle reaches a minimum speed if its path is given by $\mathbf{r}(t) = \langle 3t^2, 2t, 4t^2 - t \rangle$ where t is in seconds and distance is measured in centimeters.

6. A projectile is fired with an initial speed of 400 meters per second and angle of elevation $60°$.

   a) Find the range of the projectile.
   b) Find the maximum height of the projectile.
   c) Find the speed of the projectile upon impact.

7. A ball is thrown at an angle of $45°$ to the ground. If the ball lands 70 meters away, what was the initial speed of the ball?

8. A rifle has a muzzle velocity of 200 meters per second. Find two angles of elevation that can be used to hit a target 900 meters away.

9. Find the tangential and normal components of acceleration (use your work from number 4 on 13.3):

   a) $\mathbf{r}(t) = 2t\mathbf{i} + \cos t\mathbf{j} - \sin t\mathbf{k}$ b) $\mathbf{r}(t) = e^{-t}\mathbf{i} + e^{t}\mathbf{j} - \sqrt{2}t\mathbf{k}$

10. Consider the position function $\mathbf{r}(t) = 2t\mathbf{i} + t^3\mathbf{j}$. Do the following sketches on the same plane.

    a) Sketch the curve given by $\mathbf{r}(t)$.
    b) Sketch the acceleration vector at $t = 1$ (make sure its initial point is at $\mathbf{r}(1)$).
    c) Sketch the vectors $a_T\mathbf{T}(1)$ and $a_N\mathbf{N}(1)$ (make sure the initial points are at $\mathbf{r}(1)$).

11. Find the unit tangent, principal unit normal, and binormal vectors of $\mathbf{r}(t) = \frac{1}{3}t^3\mathbf{i} - \frac{\sqrt{2}}{2}t^2\mathbf{j} - t\mathbf{k}$ at the point given by $t = 1$.

    (Note: The unit binormal vector is defined as the cross product of vectors T and N, TxN. Also, Remember that the principal unit normal vector is given by N = T'/||T'||.)