Homework Section 15.1

1. a) Approximate the volume of the solid region under the surface $z = xy^2$ and above the rectangle $R = \{(x, y) \mid 0 \le x \le 6, 0 \le y \le 4\}$. Use a Riemann sum with $m = 3$, $n = 2$, and use the upper right corner of each sub-rectangle as sample points.

   b) Repeat part (a) using midpoints as your sample points.

2. a) Approximate the volume of the solid region under the surface $z = x^2 + 2y$ and above the rectangle $R = [1, 3] \times [0, 4]$. Use a Riemann sum with $m = 2$, $n = 2$, and use the lower right corner of each sub-rectangle as sample points.
   b) Repeat part (a) using midpoints as your sample points.

3. A table of values is given for a function $f(x, y)$ defined on $R = [2, 4] \times [0, 4]$.

   a) Estimate $\iint\limits_R f(x, y)\,dA$ using midpoints with $m = n = 2$.

   b) Estimate the double integral with $m = 2$ and $n = 4$ by using the lower left corner of each sub-rectangle as sample points.

x \ y	0	1	2	3	4
1	2	1	-2	-6	-5
1.5	3	1	-4	-7	-6
2	4	3	0	-5	-8
2.5	5	5	3	-1	-4
3	7	8	6	3	0
3.5	8	10	7	4	1

4. Evaluate the double integral by first identifying it as the volume of a solid. (Example)

   a) $\iint\limits_R 4\,dA$, $R = \{(x, y) \mid -1 \le x \le 2, 1 \le y \le 5\}$

   b) $\iint\limits_R (6 - x)\,dA$, $R = \{(x, y) \mid 0 \le x \le 6, 0 \le y \le 3\}$

5. Rectangleopia is a small, rectangular country which measures 180 miles by 117 miles. The contour map below shows elevations in feet. Use midpoints with $m = n = 2$ to estimate the average elevation in Rectangleopia. (Example).