

### Homework Section 15.5

1. Calculate the mass and the center of mass of the lamina that occupies the region  $R$  and has density function  $\rho(x, y)$ .
  - a)  $R = \{(x, y) \mid -1 \leq x \leq 1, 0 \leq y \leq 2\}$ ;  $\rho(x, y) = x^2 y$
  - b)  $R$  is bounded by the parabola  $x = y^2$  and the line  $y = -x + 2$ ;  $\rho(x, y) = 4$
2. A lamina is given by the quarter disk  $x^2 + y^2 \leq 4$  in the first quadrant. Calculate its center of mass when the density at any point on the lamina is proportional to its distance from the origin. [Hint: using polar coordinates, the distance to the origin is  $r$ .]
3. Find the moments of inertia  $I_x$ ,  $I_y$ ,  $I_0$  for the lamina in exercise 2.
4. Suppose a lamina corresponds to the region between the curve  $y = \cos x$  and the  $x$ -axis from  $x = 0$  to  $x = \pi/2$ . Given that the lamina has constant density  $\rho(x, y) = \rho$ , Calculate the moments of inertia  $I_x$  and  $I_y$  and the radii of gyration  $\bar{\bar{x}}$  and  $\bar{\bar{y}}$ .