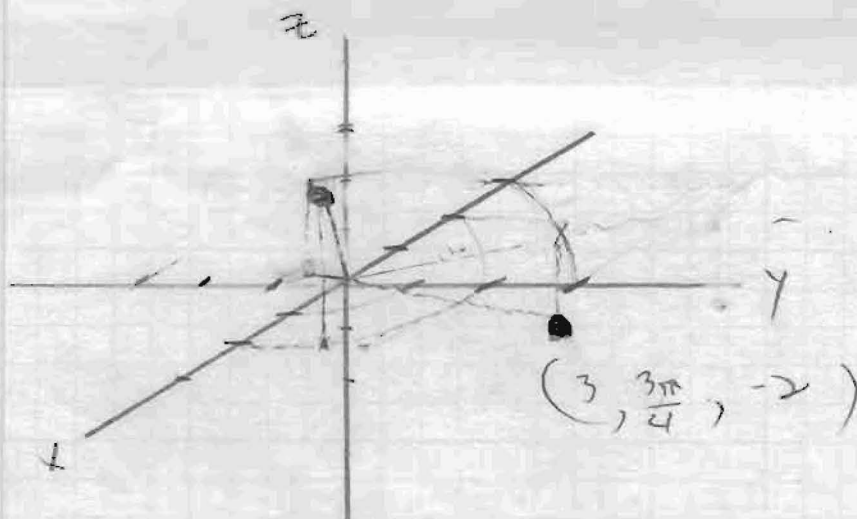


15.7 Answers

① a) $(2, \frac{\pi}{6}, 3)$

b) $(3, \frac{3\pi}{4}, -2)$



② a) $x = r \cos \theta = 2 \cos \frac{\pi}{6} = \sqrt{3}$
 $y = 2 \sin \frac{\pi}{6} = 1$

$(\sqrt{3}, 1, 3)$

b) $x = 3 \cos \frac{3\pi}{4} = -\frac{3\sqrt{2}}{2}$

$(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, -2)$

$y = 3 \sin \frac{3\pi}{4} = \frac{3\sqrt{2}}{2}$

③ $r^2 = 1 + 3 = 4$

$r = 2$

$\tan \theta = -\sqrt{3} \Rightarrow \theta = \frac{5\pi}{3}$

$(2, \frac{5\pi}{3}, 5)$

4

a) $r = 4$
 $r^2 = 16$

$x^2 + y^2 = 16$
cylinder with
axis the z -axis

b) $\theta = \frac{\pi}{6}$

$\tan \theta = \frac{\sqrt{3}}{3}$

$\frac{y}{x} = \frac{\sqrt{3}}{3}$

$y = \frac{\sqrt{3}}{3}x$, $+70$
half-plane (since
 $r \geq 0$) containing
 z -axis.

c) $r = 2 \sin \theta$

$r^2 = 2r \sin \theta$

$x^2 + y^2 = 2y$

$x^2 + y^2 - 2y + 1 = 0 + 1$

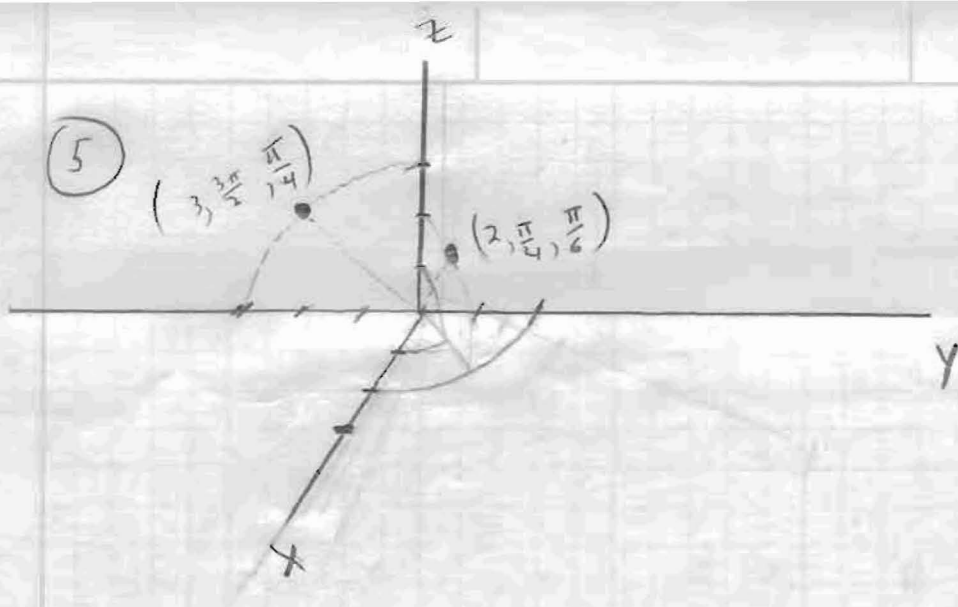
$x^2 + (y - 1)^2 = 1$

circular cylinder with radius 1 and axis
on the ^{vertical} line: $x = 0, y = 1, z = z$.

d) $r^2 - z^2 = 1$

$x^2 + y^2 - z^2 = 1$

Hyp-er-boloid of one sheet with
axis the z -axis



6) a)

$$x = \rho \sin \phi \cos \theta = 2 \sin \frac{\pi}{6} \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$y = \rho \sin \phi \sin \theta = 2 \sin \frac{\pi}{6} \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$z = \rho \cos \phi = 2 \cos \frac{\pi}{6} = \sqrt{3}$$

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \sqrt{3} \right)$$

b)

$$x = 0$$

$$y = 3 \sin \frac{\pi}{4} \sin \frac{3\pi}{2} = 3 \frac{\sqrt{2}}{2} (-1) = -\frac{3\sqrt{2}}{2}$$

$$z = 3 \cos \frac{\pi}{4} = \frac{3\sqrt{2}}{2}$$

$$\left(0, -\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \right)$$

7) $(1, -1, \sqrt{3})$

$$\rho = \sqrt{1 + 1 + 3} = \sqrt{5}$$

$$\left(\sqrt{5}, \frac{7\pi}{4}, \cos^{-1} \left(\frac{\sqrt{3}}{5} \right) \right)$$

$$\tan \theta = -1 \Rightarrow \theta = \frac{7\pi}{4}$$

$$z = \rho \cos \phi \Rightarrow \sqrt{3} = \sqrt{5} \cos \phi$$

$$\frac{\sqrt{3}}{\sqrt{5}} = \cos \phi$$

$$\phi = \cos^{-1} \left(\frac{\sqrt{3}}{5} \right)$$

(8) a) $\rho = 4 \Rightarrow \rho^2 = 4^2 \Rightarrow x^2 + y^2 + z^2 = 16$
 sphere, radius = 4

9a b) $\rho \sin \phi = 4$
 $r = 4$

$$\sqrt{x^2 + y^2} = 4$$

$$x^2 + y^2 = 16$$

circular cylinder
 of radius 4 about
 z-axis

9b c) $\rho = \sin \phi \cos \theta$

$$\rho^2 = \rho \sin \phi \cos \theta$$

$$x^2 + y^2 + z^2 = x$$

$$x^2 - x + \frac{1}{4} + y^2 + z^2 = 0 + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + y^2 + z^2 = \frac{1}{4}$$

sphere, center $\left(\frac{1}{2}, 0, 0\right)$

$$r = \frac{1}{2}$$

8b d) $\phi = \pi/4$

$$\rho \cos \phi = \rho \frac{1}{\sqrt{2}}$$

$$z = \frac{1}{\sqrt{2}} \sqrt{x^2 + y^2 + z^2}$$

$$2z^2 = x^2 + y^2 + z^2$$

$$z^2 = x^2 + y^2$$

Top half of a cone
 with axis the z-axis

$$(10) a) z = 2x^2 + 2y^2$$

$$z = 2(x^2 + y^2)$$

$$z = 2r^2$$

$$\rho \cos \phi = 2\rho^2 \sin^2 \phi$$

$$\cos \phi = 2\rho \sin^2 \phi$$

$$b) r^2 + z^2 = 16$$

$$\rho^2 = 16$$

$$\rho = 4$$

$$c) r^2 = 3r \sin \theta$$

$$r = 3 \sin \theta$$

$$\rho \sin \phi = 3 \sin \theta$$