Homework Section 15.8

1. Sketch the solid whose volume is given by the integral:

   a) $\int_0^3 \int_0^{2\pi} \int_r^3 r \, dz d\theta dr$ b) $\int_0^{\pi/2} \int_0^2 \int_0^{4-r^2} r \, dz dr d\theta$

   c) $\int_0^{\pi/2} \int_0^{\pi} \int_0^2 \rho^2 \sin\phi \, d\rho d\theta d\phi$

2. Use cylindrical coordinates to evaluate the triple integrals:

   a) $\iiint_Q (x^2 y + y^3) dV$, where Q is the solid in the first octant that lies beneath the paraboloid $z = 9 - x^2 - y^2$

   b) $\iiint_Q x dV$, in the first octant where Q is enclosed by the planes, $z = 0$, $z = y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

   c) $\iiint_Q y^2 dV$, where Q is the solid that lies within the cylinder $x^2 + y^2 = 4$, above the *xy*-plane, and below the half-cone $z = \sqrt{4x^2 + 4y^2}$.

3. Use Spherical Coordinates to evaluate the triple integrals:

   a) $\iiint_Q (x^2 + y^2 + z^2) dV$, where Q is the spherical solid $x^2 + y^2 + z^2 \le 25$.

   b) $\iiint_Q x dV$, where Q lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ in the first octant.

   c) $\iiint_Q z \, dV$, where Q lies between the spheres $\rho = 1$ and $\rho = 4$ and inside the cone $\phi = \pi/6$.

4. Find the volume of the solid that lies inside the sphere $x^2 + y^2 + z^2 = 6$, and below the half-cone $z = \sqrt{x^2 + y^2}$.

5. Find the volume of the smaller wedge cut from a sphere of radius *a* by two planes that intersect along a diameter at an angle of $\pi/3$.

6. Evaluate the integral by changing to cylindrical coordinates: $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} 2yz \; dzdxdy$.

7. Evaluate the integral by changing to spherical coordinates: $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z(x^2+y^2+z^2) \; dzdydx$

8. a) Set up **and** evaluate a triple integral in cylindrical coordinates that gives a formula for the volume of a sphere of radius *a*.

   b) Repeat part (a) using spherical coordinates.