

### Homework Section 16.3

1. A table of values of a function  $f$  with continuous gradient is given below.

Evaluate  $\int_C \nabla f \cdot d\mathbf{r}$ , where  $C$  is a smooth curve beginning at  $(0, 0)$  and ending at  $(2, 1)$ .

$x \backslash y$	<b>0</b>	<b>1</b>	<b>2</b>
<b>0</b>	-1	6	5
<b>1</b>	3	4	7
<b>2</b>	8	3	9

2. Determine whether or not  $\mathbf{F}$  is a conservative vector field. If it is, find a function  $f$  such that  $\mathbf{F} = \nabla f$ .

a)  $\mathbf{F}(x, y) = (5x + 3y)\mathbf{i} + (3x + 5y)\mathbf{j}$

b)  $\mathbf{F}(x, y) = (x^3 + 2xy)\mathbf{i} + (4xy - 2x)\mathbf{j}$

c)  $\mathbf{F}(x, y) = (ye^x + \cos y)\mathbf{i} + (e^x + x \sin y)\mathbf{j}$

3. Let  $C$  be a piecewise smooth curve in an open region that goes from the point  $(2,3)$  to the point  $(5,6)$ , and  $f(x, y) = x^2y$  is a potential function for a continuous vector field  $\mathbf{F}(x, y)$ . Find the work done by  $\mathbf{F}(x, y)$  on an object moving along the path from  $(2,3)$  to  $(5,6)$ .

4. Find a function  $f$  such that  $\mathbf{F} = \nabla f$  and use it to evaluate  $\int_C \nabla f \cdot d\mathbf{r}$  along the given curve  $C$ .

a)  $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}$ ,  $C$  is the line segment from  $(0, 1, -3)$  to  $(5, 4, 3)$

b)  $\mathbf{F}(x, y, z) = y^2 \sin z\mathbf{i} + 2xy \sin z\mathbf{j} + xy^2 \cos z\mathbf{k}$ ,  $C: \mathbf{r}(t) = t^2\mathbf{i} + \cos t\mathbf{j} + t\mathbf{k}$ ,  $0 \leq t \leq \pi$