Homework Section 16.6

1. Determine whether the points P(-2, 6, 5) and Q(7, 3, 15) lie on the surface given by $\mathbf{r}(u,v) = \langle u + 3v, 1 + 4u - v, 5 + u + v \rangle$.

2. Find a Cartesian equation for the given parametric surface and identify it.

   a) $\mathbf{r}(u,v) = (u - v)\mathbf{i} + (2 - v)\mathbf{j} + (4 + 2u - 4v)\mathbf{k}$

   b) $\mathbf{r}(s,t) = s\mathbf{i} + t\mathbf{j} + (s^2 + t^2)\mathbf{k}$

   c) $\mathbf{r}(u,v) = \langle u, u\sin 3v, u\cos 3v \rangle$

3. Find a parametric representation for the surface.

   a) The hyperbolic paraboloid $z = x^2 - 2y^2$.

   b) The lower half of the ellipsoid $x^2 + 4y^2 + 9z^2 = 1$.

   c) The sphere $x^2 + y^2 + z^2 = 9$.

   d) The cylinder $x^2 + y^2 = 4$ from $z = 0$ and $z = 3$.

4. Find an equation of the tangent plane to the given parametric surface at the specified point: $\mathbf{r}(u,v) = u\sin v\mathbf{i} + u^2\mathbf{j} + 2u\cos v\mathbf{k}$; $u = 1$, $v = \pi$.

5. Find the area of the surface.

   a) The part of the plane $2x + y + z = 6$ that lies in the first octant.

   b) The part of the hyperbolic paraboloid $f(x,y) = x^2 - y^2$ that lies within the cylinder $x^2 + y^2 = 4$.

   c) The part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 4$.

   d) The surface given by $\mathbf{r}(u,v) = uv\mathbf{i} + v^2\mathbf{j} + \frac{1}{2}v^2\mathbf{k}$, where $0 \le u \le 2$, $0 \le v \le 1$.