

Homework Section 16.7

1. Evaluate the surface integral.

a) $\iint_S z dS$, S is the part of the plane $x + y + z = 2$ that lies in the first octant.

b) $\iint_S y^2 z^2 dS$, S is the part of the cone $f(x, y) = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 4$.

c) $\iint_S x dS$, S is the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 4$. (**Hint:** use the rectangular form of dS with $x = g(y, z)$).

d) $\iint_S (x^2 + y^2 + z^2) dS$, S is the part of the cylinder $x^2 + y^2 = 4$ between the planes $z = 0$ and $z = 3$. (**Hint:** parameterize the cylinder and use the parametric form of dS).

2. Evaluate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{N} dS$.

a) $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$, S is the part of the plane $x + y + z = 2$ that lies in the first octant, and has upward orientation.

b) $\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + \mathbf{k}$, S is part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the square $0 \leq x \leq 1$, $0 \leq y \leq 1$, and has upward orientation.

c) $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 4$ with downward orientation.

d) $\mathbf{F}(x, y, z) = -x\mathbf{i} + z\mathbf{k}$, S consists of the paraboloid $z = f(x, y) = x^2 + y^2$, $0 \leq z \leq 1$, **and** the disk $x^2 + y^2 \leq 1$, $z = 1$. [**Note:** this requires evaluation of two integrals, and the paraboloid should be oriented downwards]

e) $\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$, S is the part of the sphere $x^2 + y^2 + z^2 = 9$ in the first octant with orientation toward the origin (**Hint:** use the parametric form of the flux integral).