Homework Section 16.8

1. Use a line integral to evaluate $\iint_S (\textbf{curl F}) \bullet \mathbf{N} dS$, where $\mathbf{F}(x, y, z) = -yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ and S is the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane z = 1, oriented upward.

2. Use a surface integral (i.e. the right-hand side of Stokes' Theorem) to evaluate $\int_C F \bullet d\mathbf{r}$. In each case C is oriented counter-clockwise (as viewed from above) and bounds a surface, S.

   a) $\mathbf{F}(x, y, z) = (x + z^2)\mathbf{i} + (y + x^2)\mathbf{j} + (z + y^2)\mathbf{k}$, C is the triangle with vertices (1,0,0), (0,1,0) and (0,0,2).

   b) $\mathbf{F}(x, y, z) = e^y\mathbf{i} + e^{-y}\mathbf{j} + e^z\mathbf{k}$, C is the boundary of the plane $x + 3y + 3z = 3$ in the first octant.

   c) $\mathbf{F}(x, y, z) = y\mathbf{i} - x^2\mathbf{j} + z^2\mathbf{k}$ and C is the curve of intersection of the plane $x + z = 1$ and the cylinder $x^2 + y^2 = 1$.

3. Consider two surfaces, $S_1 : z = \sqrt{1 - x^2 - y^2}$ and $S_2 : z = 1 - x^2 - y^2$, bounded below by the xy-plane. Suppose **F** is a vector field over R^3 whose components have continuous partial derivatives. Explain why $\iint_{S_1} (\textbf{curl F}) \bullet \mathbf{N} dS = \iint_{S_2} (\textbf{curl F}) \bullet \mathbf{N} dS$.