

# Hyperbolas

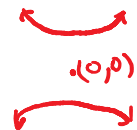
Goal: To graph hyperbolas

## The Standard Equations of Hyperbolas Centered at (0,0)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

 opens horizontally

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

 opens vertically

Every hyperbola has two asymptotes.  
Solve for  $y$  to find equations of asymptotes:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$-1 \quad + \frac{y^2}{b^2} \quad - \quad \frac{y^2}{b^2}$

$$\frac{x^2}{a^2} - 1 = \frac{y^2}{b^2}$$

$$\frac{x^2}{b^2} - \frac{y^2}{b^2} = b^2 \left( \frac{x^2}{a^2} - 1 \right)$$

$$\sqrt{y^2} = \pm \sqrt{\frac{b^2}{a^2} x^2 - b^2}$$

$$y = \pm \sqrt{\frac{b^2}{a^2} x^2 - b^2}$$

$$y \approx \pm \sqrt{\frac{b^2}{a^2} x^2} \quad \text{for "large" } x^2$$

$$y \approx \pm \frac{b}{a} x$$

$$y = \pm \frac{b}{a} x$$

equations of asymptotes.

opens horizontally

asymptote eqns.  
when hyperbola  
opens vertically

$$y = \pm \frac{a}{b} x$$

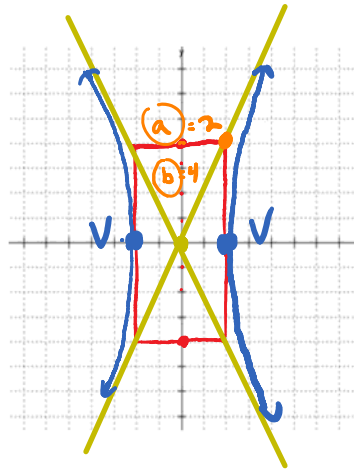
Opens vertically

(ex) Graph the hyperbola:

a)  $\frac{x^2}{4} - \frac{y^2}{16} = 1$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a^2 = 4 \rightarrow a = 2$   
 $b^2 = 16 \rightarrow b = 4$

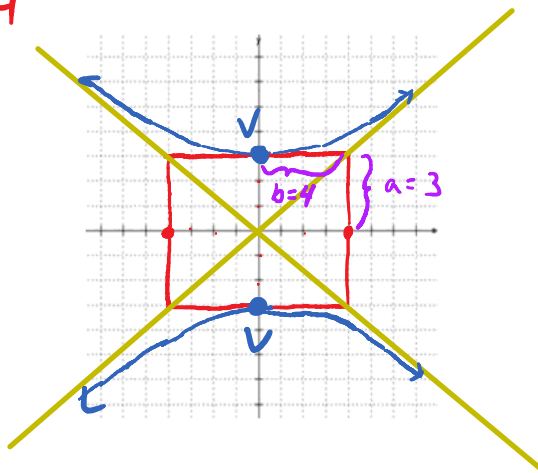


b)  $\frac{16y^2}{144} - \frac{9x^2}{144} = \frac{144}{144}$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$a = 3, b = 4$

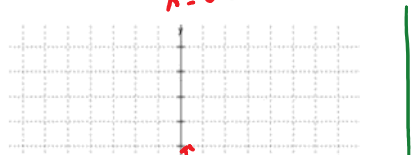


c)  $xy = 1$

$\frac{xy}{x}$	$=$	$\frac{1}{x}$
$\frac{1}{x}$	$=$	$\frac{1}{x}$

x	y
-2	$-\frac{1}{2}$
-1	-1

$x=0 \leftarrow$  asymptotes



$\frac{1}{x}$   $\frac{1}{x}$

$y = \frac{1}{x}$

rational fn.  $\leftarrow$

-2	$-\frac{1}{2}$
-1	-1 $\leftarrow$
$-\frac{1}{2} = -0.5$	-2
0	$\frac{1}{0} = \phi$
$\frac{1}{2} = 0.5$	2
1	1
2	$\frac{1}{2}$

$y = \frac{1}{x} = \frac{1}{2}$

