The Algebra of Functions

Goals: to add, subtract, multiply, and divide functions.

ex Let $f(x) = 2x+8$ and $g(x) = \boxed{-x^2+3x+5}$ Find...

a) $(f+g)(2)$

$f+g$ = gives another function

$(f+g)(2) \overset{def}{=} f(2)+g(2)$

$f(2)$

$f(2) = 2(2)+8$

$= 4+8$

$= 12$

$g(2) = -(2)^2 + 3(2) + 5$

$= \underbrace{-4 + 6}_{2+5} + 5$

$= 7$

$(f+g)(2) = 12 + 7 = 19$

b) $(f-g)(x)$

$\overset{def}{=} \underbrace{f(x)}_{} - \underbrace{g(x)}_{}$

$= (2x+8) - (-x^2+3x+5)$

$(f-g)(2) = f(2)-g(2)$

$= 12-7$

$= 5$ ✓

$= 2x + 8 + x^2 - 3x - 5$

$= x^2 - x + 3 \longrightarrow (f-g)(2) = 2^2 - 2 + 3$

$= 2 + 3$

$= 5$ ✓

c) $(f \cdot g)(2)$

$\overset{def}{=} f(2) \cdot g(2)$

$= 12 \cdot 7$

$= 84$

Let $\boxed{f(x) = 2x + 8}$ and $g(x) = \boxed{-x^2 + 3x + 5}$.

d) $\left(\frac{f}{g}\right)(1)$

$\overset{def}{=} \dfrac{f(1)}{g(1)}$

$f(1) = 10$, $g(1) = 7$

$= \dfrac{10}{7}$

ex Let $f(x) = 5x^2$, $g(x) = \frac{1}{x-4}$, $h(x) = x+6$.

Find the domain of ...

a) $f+h$

$$(f+g)(x) = f(x)+g(x)$$
$$= 5x^2 + x + 6$$

all reals

$f(x) = 5x^2$

any #

Dom f is all reals

$h(x) = x+6$

Dom h is all reals

b) f/h

$$\frac{5x^2}{x+6}$$

$x+6=0$

$x=-6$

$\{x \mid x \neq -6\}$

c) $f \cdot g$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$= \frac{5x^2}{1} \cdot \frac{1}{x-4}$$
$$= \frac{5x^2}{x-4}$$

Notes: ① The domain of $f+g$, $f-g$, and $f \cdot g$ is the set of elements common to the domains of both f and g.

(2) The domain of f/g is also restricted to the elements common to the domains of both f and g such that $g \neq 0$.