The Algebra of Functions

Goals: to add, subtract, multiply, and divide functions.
(ex) Let $f(x)=2 x+8$ and $g(x)=-x^{2}+3 x+5$ Find...

$$
\begin{aligned}
& \text { a) }(f+g)(2) \\
& (f+g)(2) \stackrel{\text { def }}{=} f(2)+g(2) \\
& f+g=\text { gives another } \\
& \text { function } \\
& f(2) \\
& f(2)=2(2)+8 \quad g\left(\frac{\downarrow}{2}\right)=-(2)^{2}+3(2)+5 \\
& \begin{array}{l}
=4+8 \\
=12
\end{array} \\
& \begin{array}{l}
=-\underbrace{2+5}+5 \\
=7)^{2+5}+5
\end{array} \\
& (f+g)(2)=12+7=(19)
\end{aligned}
$$

b) $(f-g)(x)$
def

$$
\begin{aligned}
& =\underbrace{f(x)}_{\downarrow}-\underbrace{g(x)}_{-x} \\
& =(2 x+8)-\underbrace{(-x}_{(-x+3 x+5)}+
\end{aligned}
$$

$$
\begin{aligned}
(f-g)(2) & =f(2)-g(2) \\
& =12-1 \\
& =5
\end{aligned}
$$

$$
\left.\begin{array}{rl}
=\left(x^{2}-x+8+4^{2}\left(-\frac{3}{x}-5\right.\right. \\
= & (f-g)(2)
\end{array}\right) 2^{2}-2+3
$$

c) $(f \cdot s)(2)$
def

$$
\begin{aligned}
& =f(2) \cdot g(2) \\
& =12 \cdot 7 \\
& =84
\end{aligned}
$$

Let $f(x)=2 x+8$ and $g(x)=-x^{2}+3 x+5$
d) $\left(\frac{f}{g}\right)(1)$
def

$$
\begin{aligned}
& =\frac{f(1)}{s(1)} \\
& =\frac{10}{7}
\end{aligned}
$$

$$
f(1)=10 \quad, g(1)=7
$$

(ex) Let $f(x)=5 x^{2}, g(x)=\frac{1}{x-4}, \quad h(x)=x+6$. Find the domain of ...
a) $f+h$

$$
\begin{aligned}
(f+g)(x) & =f(x)+g(x) \\
& =5 x^{2}+x+6
\end{aligned}
$$

all reals

$$
f(x)=5 x^{2}
$$

Dome is all reals

$$
h(x)=x+6
$$

$(x)=x+6$
Dom $h$ is all reals
b) $f / h$
$x+6$

$$
\{x \mid x \neq-6\}
$$

$$
\begin{aligned}
x+6 & =0 \\
x & =-6
\end{aligned}
$$

C) $f .9$

$$
\begin{aligned}
(f \cdot g)(x) & =f(x) \cdot g(x) \\
& =\frac{5 x^{2}}{1} \cdot \frac{1}{x-4} \\
& =\frac{5 x^{2}}{x-4}
\end{aligned}
$$

Notes: (1) The domain of $f+g, f-g$, and $f \cdot g$ is the set of elements common to the domains of " $f$ both $g$.
(2) The domain of $\mathrm{f} / \mathrm{g}$ is also restricted to the elements common to the domains of ${ }^{\text {both }} f$ and $S$ such that $g \neq 0$.

