Solving Systems of Three Equations and Three Unknowns

Goal: Let's solve these things!

(ex) Solve

E_1 $3x + 2y + 3z = 3$

E_2 $4x - 5y + 7z = 1$

E_3 $2x + 3y - 2z = 6$

① select 2 eqns., eliminate 1 unknown.

$-2E_3 + E_2$

$-4x - 6y + 4z = -12$

$4x - 5y + 7z = 1$

$-11y + 11z = -11$

$\frac{-11y}{-11} + \frac{11z}{-11} = \frac{-11}{-11}$

$y - z = 1$

② Select the un-used eqn. and one of the others and eliminate same unknown

$-2E_1 + 3E_3$

$-6x - 4y - 6z = -6$

$6x + 9y - 6z = 18$

$5y - 12z = 12$

③ Solve resulting system of 2 eqns., 2 unknowns

$-5[y - z = 1]$

$5y - 12z = 12$

$-5y + 5z = -5$

$-7z = 7$

$z = -1$

$y - (-1) = 1$

$y + 1 = 1$

$y = 0$

④ Plug back into one of the original eqns. to find the last unknown

$3x+2y+3z=3$
$4x-5y+7z=1$
$2x+3y-2z=6$

$3x+2(0)+3(-1)=3$

$3x-3=3$

$3x=6$

$x=2$

$(2, 0, -1)$ ← represents the intersection of 3 planes in space given by the original eqns.

b) E_1 $x+y+z=5$

E_2 $x-y+z=1$

E_3 $x-z=y+3$

$-y \quad -y$

E_1 $x+y+z=5$

E_2 $x-y+z=1$

E_3 $x-y-z=3$

① E_1+E_3

$2x=8$

$x=4$

② E_2+E_3

$2x-2y=4$

③

$8-2y=4$

$-2y=-4$

$y=2$

$4+2+z=5$

$6+z=5$

$z=-1$

$(4, 2, -1)$

c) E_1 $x-3y+z=4$

E_2 $x+5y-z=2$

E_3 $-2x+2y-z=-1$

① E_1+E_2

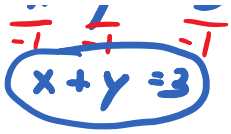
$2x+2y=6$

$x+y=3$

② E_1+E_3

$-x-y=-3$

$x+y=3$



(3) $-1(x+y=3)$

$x + y = 3$

$-x - y = -3$

$0 = 0$ True

dependent

So there are infinitely many solutions