Absolute Value Equations and Inequalities

Goal: To solve these things!

$$|-5| = 5$$

$$|5| = 5$$

Absolute value measures distance from 0.

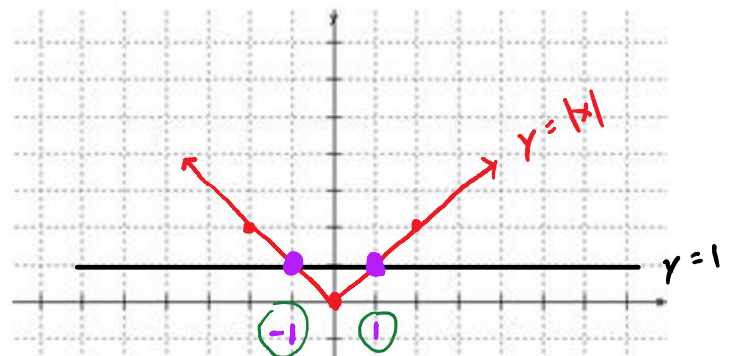
(ex) Solve $|x| = 1$

$$x = -1 \text{ or } x = 1 \checkmark$$

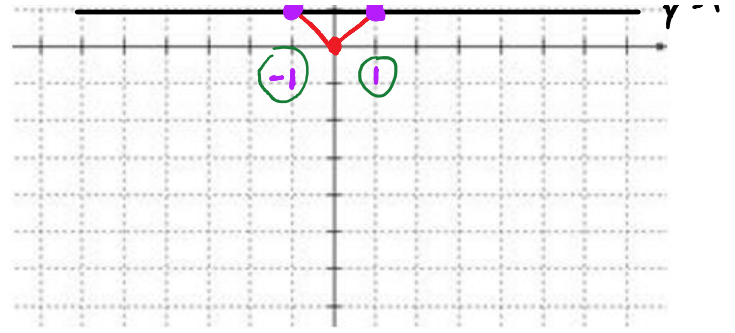(ex) Solve $\boxed{|x| = 1}$ by graphing.

$y_1 \qquad y_2$

$\bigcirc{y} = |x| \qquad \bigcirc{y} = 1$

x	y
-2	2

$y = |x|$

$y = 1$

$(-1) \quad (1)$

$$\begin{array}{c|c} -2 & 2 \\ -1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 2 \end{array}$$

$$\boxed{x = 1, \; x = -1}$$

(ex) Solve the absolute value equation

a) $|3x - 1| = 5$

$3x - 1 = -5$ or $3x - 1 = 5$

$\quad\quad +1 \quad +1$

$\dfrac{3x}{3} = \dfrac{-4}{3}$ $\qquad$ $3x = 6$

$x = \dfrac{-4}{3}$ ✓   or $x = 2$ ✓ $\qquad \left\{ \dfrac{-4}{3}, 2 \right\}$

b) $5|q| - 2 = 9$ $\qquad$ ★ Isolate abs. value first

$\quad\quad\quad +2 \quad +2$

$\dfrac{5|q|}{5} = \dfrac{11}{5}$

$|q| = \dfrac{11}{5}$

$$|q| = \frac{11}{5}$$

$$q = \frac{-11}{5} \quad \text{or} \quad g = \frac{11}{5}$$

c) $|3a - 1| = |2a + 4|$ $|-5| = |5|$

$3a - 1 = -(2a + 4)$ or $\begin{aligned} 3a - 1 &= 2a + 4 \\ -2a + 1 \quad &-2a + 1 \end{aligned}$

$3a - 1 = -2a - 4$ or $a = 5$

$5a = -3$

$a = \frac{-3}{5}$ or $a = 5$

$a = \frac{-3}{5}$ or $a = 5$

(ex) solve the absolute value inequality $|x| < 1$.

① solve related equation to get critical values

$$|x| = 1$$

$x = -1$ or $x = 1$ cv's

② Plot cv's on a number line

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NO!
TV = -2

yes
TV = 0

NO!
TV = 2

$-\infty$ X -1 1 ∞
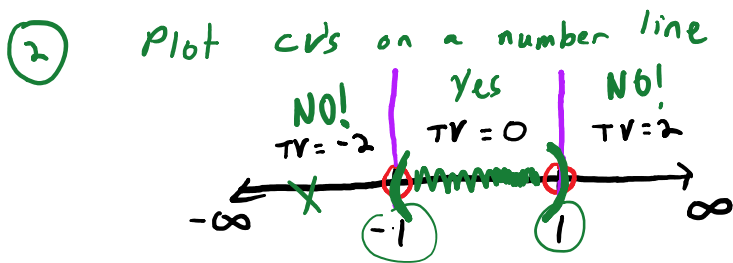
③ use test values to determine solution interval(s)

$|x| < 1$

TV = -2

TV = 0

TV = 2

$|-2| \overset{?}{<} 1$
NO!

$|0| \overset{?}{<} 1$ Yes!

$|2| \overset{?}{<} 1$ NO!

$(-1, 1)$

ex   Solve $|x| < 1$ by graphing.
         Y   Y

$y = |x|$

$y = 1$

$y = |x|$

$y = 1$

-1 1
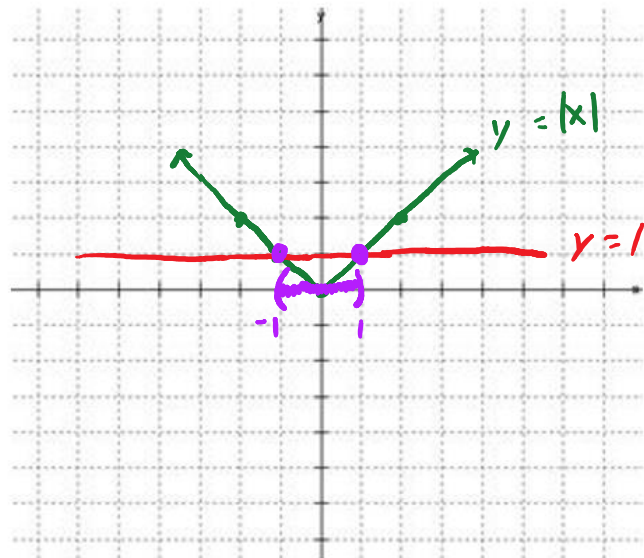
(ex) Solve the given inequality.

a) $|1 + 5x| - 2 < 4$

$+2 \quad +2$

$|1 + 5x| \boxed{<} 6$

① $|1 + 5x| = 6$

$1 + 5x = -6$ or $1 + 5x = 6$

-1

$5x = -7 5x = 5$

$\overline{5} \quad \overline{5}$

CVs

$x = -\dfrac{7}{5}$ or $x = 1$

②

TV$=-2$ TV$=0$

$-\infty -\dfrac{7}{5} 1 \infty$

$\boxed{|1 + 5x| < 6}$

③ TV$= -2$ TV$= 0$ TV$= 2$

$|1 + 5(-2)| \overset{?}{<} 6$ $|1 + 5(0)| \overset{?}{<} 6$ $|1 + 5(2)| \overset{?}{<} 6$

$|-9| \overset{?}{<} 6$ Yes NO!
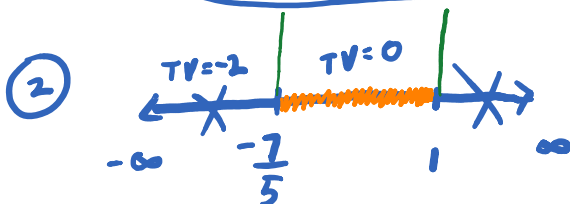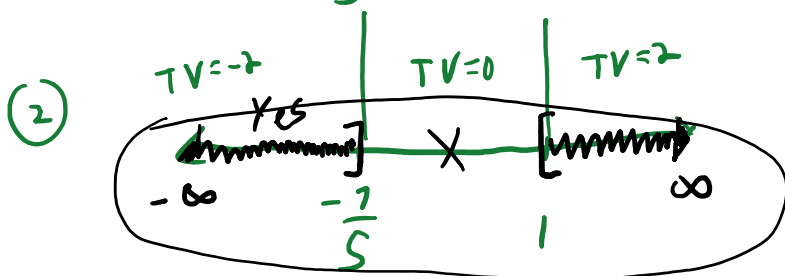
NO!

$\boxed{\left(-\dfrac{7}{5}, 1\right)}$

b) $\dfrac{2 \cdot |1 + 5x|}{2} \geq \dfrac{12}{2}$

$$|1+5x| \geq 6$$

① $|1+5x| = 6$

$1 + 5x = -6$ or $1 + 5x = 6$

$x = \dfrac{-7}{5}$ or $x = 1$

②

TV=-2   TV=0   TV=2

Yes   X

$-\infty$ $\dfrac{-7}{5}$ 1 ∞

$|1+5x| \geq 6$

TV = -2

$|1 + 5(-2)| \overset{?}{\geq} 6$

$9 \geq 6$

Yes

TV = 0

$|1 + 5(0)| \geq 6$

No!

TV = 2

$|1 + 5(2)| \geq 6$

$11 \geq 6$

yes

$$\left(-\infty, \dfrac{-7}{5}\right] \cup [1, \infty)$$