Even More Fun with Factoring

Goal: To factor polynomials that are the sum or difference of two cubes.

Perfect Cubes

$$\boxed{\begin{array}{ll} 1^3 = 1 & 7^3 = 343 \\ 2^3 = 8 & \vdots \\ 3^3 = 27 & \\ 4^3 = 64 & \\ 5^3 = 125 & \\ 6^3 = 216 & \end{array}}$$

The Sum/Difference of Two Cubes Formulas

$$\boxed{\begin{aligned} A^3 + B^3 &= (A+B)(A^2 - AB + B^2) \\ A^3 - B^3 &= (A-B)(A^2 + AB + B^2) \end{aligned}}$$

← memorize!

proof: $(A+B)(A^2 - AB + B^2) = A^3 \cancel{- A^2B} + \cancel{AB^2} + B^3$

$\qquad\qquad\qquad\qquad\qquad\quad\; \cancel{+ A^2B} \cancel{- AB^2}$

$= A^3 + B^3$

Done

ex Factor

a) $x^3 - 27$ $\qquad A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A = x, \quad B = 3$

$= \boxed{(x-3)(x^2 + 3x + 9)}$

b) $16p^3 + 2q^3$ $\qquad A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

$2(8p^3 + q^3)$ $\qquad \boxed{A = 2p}, \quad B = q$

$(2p)^3 = 2p \cdot 2p \cdot 2p = 8p^3$

$= 2^3 p^3$

$= 8p^3$

$2(2p+q)((2p)^2 - 2pq + q^2)$

$\boxed{2(2p+q)(4p^2 - 2pq + q^2)}$

c) $\underbrace{125c^6}_{A^3} - \underbrace{\frac{1}{8}d^3}_{B^3}$ $\qquad A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A = 5c^2, \quad B = \frac{1}{2}d$

$(5c^2 - \frac{1}{2}d)((5c^2)^2 + \frac{5}{2}c^2 d + (\frac{1}{2}d)^2)$

$\boxed{(5c^2 - \frac{1}{2}d)(25c^4 + \frac{5}{2}c^2 d + \frac{1}{4}d^2)}$

d) $\underbrace{x^6}_{A^2} - \underbrace{64y^6}_{B^2}$

$A^2 - B^2 = (A+B)(A-B)$

$A = x^3 \quad B = 8y^3$

$= \underbrace{\overbrace{(x^3+8y^3)}^{\text{sum of 2 cubes}}} \underbrace{\overbrace{(x^3-8y^3)}^{\text{diff. of 2 cubes}}}$

$A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$

$\boxed{A = x,\ B = 2y}$

$= \boxed{(x+2y)(x^2-2xy+4y^2)(x-2y)(x^2+2xy+4y^2)}$

Factoring Guidelines (section 5.7)

1. Factor out GCF first, if necessary
2. Look at the number of terms.
   a) Two terms - try difference of two squares first, sum/diff. of two cubes 2nd.
   b) Three terms - try reverse foil.
   c) Four terms - try factoring by grouping.

c) Four terms - try factoring by grouping.

③ Factor completely.

④ Check by multiplying back out.