Polynomial Equations

Goal: To solve polynomial equations and applications of polynomial equations.

Zero Product Rule

If $\underbrace{A \cdot B} = 0$, Then $A = 0$ or $B = 0$

ex solve

a) $\underbrace{(x+3)}_{A}\underbrace{(x-7)}_{B} = 0$

$x+3=0$ or $x-7=0$

★ $x = -3$ or $x = 7$

$\{-3, 7\}$

b) $6t^2 = 8t$

$-8t \quad -8t$

$\underbrace{AB} = 0 \leftrightarrow A = 0$ or $B = 0$

Check

$0 = 0$ ✓

1 $6t^2 - 8t = 0$

5

$$6t^2 - 8t = 0$$

(1) $6t^2 - 8t = 0$

(2) $2t(3t-4) = 0$

(3) $2t = 0$ or $3t - 4 = 0$

(4) $t = 0$ or $t = \frac{4}{3}$

(5) Check

$0 = 0$ ✓

$6\left(\frac{4}{3}\right)^2 = \frac{8}{1}\left(\frac{4}{3}\right)$

$\frac{6}{1} \cdot \frac{16}{9} = \frac{32}{3}$ ✓

c) $9a + a^2 = -20$

$+20 \quad +20$

(1) $1a^2 + 9a + 20 = 0$

(2) $(a+4)(a+5) = 0$

(3) $a+4 = 0$ or $a+5 = 0$

(4) $a = -4$ or $a = -5$

(5) ✓

Steps

(1) set eqn. $= 0$

(2) factor

(3) Apply ZPR (set factors $= 0$)

(4) solve

(5) check

d) $x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x+4 = 0$ or $x - 4 = 0$

$A^2 - B^2 = (A+B)(A-B)$

$x + 4 = 0$ or $x - 4 = 0$

$x = -4$ or $x = 4$

$x = \pm 4$

e) $(y - 6)(y + 6) = 45$

$-6y$

$+6y$

$y^2 - 36 = 45$

$-45 \quad -45$

$y^2 - 81 = 0$

$(y + 9)(y - 9) = 0$

$y + 9 = 0$ or $y - 9 = 0$

$y = -9$ or $y = 9$

$A^2 - B^2 = (A + B)(A - B)$

$A = y,\ B = 9$

★ f) $2x^3 + 6x^2 - 8x - 24 = 0$

$2[x^3 + 3x^2 - 4x - 12] = 0$

$2[x^2(x + 3) - 4(x + 3)] = 0$

$$2\left[\underbrace{(x^2-4)}\,(x+3)\right]=0$$

$$2\,0$$

$$AB=0$$

$$2\left[(x+2)(x-2)(x+3)\right]=0$$

$$2(x+2)(x-2)(x+3)=0$$

$x+2=0$, $x-2=0$, $x+3=0$

$x=-2$, $x=2$, $x=-3$

ex Let $\boxed{f(x)=3x^2-8x}$. Find all a such that $\underbrace{f(a)}=-4$

$f(a)=3a^2-8a$

$3a^2-8a=-4$

$\vdots$ solve

ex Find the domain: $f(x) = \dfrac{2}{x^2 - 7x + 10}$

$x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x = 2$ or $x = 5$

$D: \{ x \mid x \neq 2 \text{ and } x = 5 \}$

You Try It!

① Solve: $x(5 + 12x) = 28$

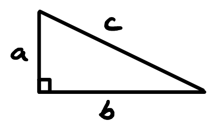
② Find the domain: $f(x) = \dfrac{x}{6x^2 - 54}$

Applications

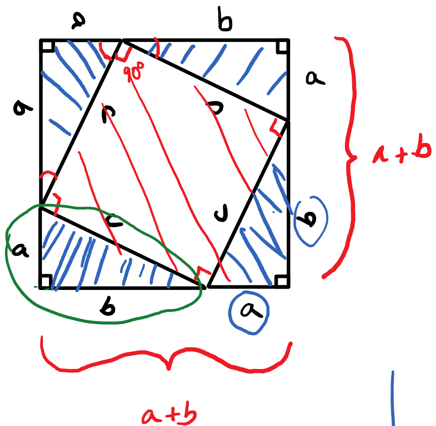
Applications

The Pythagorean Theorem

Given ⟶



conclusion: $a^2 + b^2 = c^2$



Area big Square

$$(a+b)(a+b)$$

$$(a+b)^2 = \underbrace{4 \cdot \tfrac{1}{2} ab}_{\text{area from 4 triangles}} + \underbrace{c^2}_{\text{area of inner square}}$$

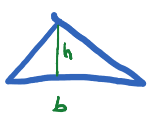
$$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$$

$$-2ab \qquad -2ab$$

$$a^2 + b^2 = c^2$$

Done

$A = \frac{1}{2} bh$



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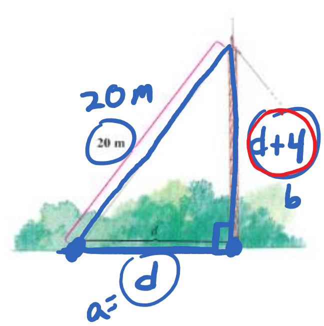
86. *Antenna wires.* A wire is stretched from the ground to the top of an antenna tower, as shown. The wire is 20 m long. The height of the tower is 4 m greater than the distance d from the tower's base to the bottom of the wire. Find the distance d and the height of the tower.



$$d^2 + (d+4)^2 = 20^2$$

$$d^2 + d^2 + 8d + 16 = 400$$

$$(A+B)^2 = A^2 + 2AB + B^2$$

$$\frac{2d^2}{2} + \frac{8d}{2} + \frac{16}{2} = \frac{400}{2}$$

$$d^2 + 4d + 8 = 200$$
$$-200 \quad -200$$

$$1d^2 + 4d - 192 = 0$$

$$(d - 12)(d + 16) = 0$$

$$d - 12 = 0 \text{ or } d + 16 = 0$$

$$d = 12\text{m} \text{ or } \cancel{d = -16}$$

tower $= 12 + 4 = 16$m.

So, the tower height is 16 m, and $d = 12$ m.

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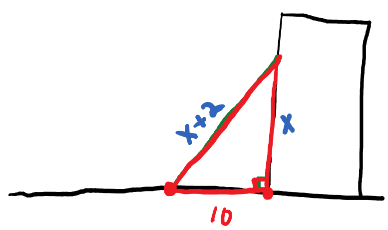
88. *Ladder location.* The foot of an extension ladder is 10 ft from a wall. The ladder is 2 ft longer than the height that it reaches on the wall. How far up the wall does the ladder reach?



$$10^2 + x^2 = (x+2)^2$$

$$10^2 + \cancel{x^2} = \cancel{x^2} + 4x + 4$$
$$-x^2 \quad -x^2$$

$$100 = 4x + 4$$
$$4x + 4 = 100$$
$$4x = 96$$
$$x = 24 \text{ ft}$$

The ladder reaches 24 ft up the wall.

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83. *Furniture.* The base of a triangular tabletop is 20 in longer than the height. The area is 750 in^2. Find the height and the base.

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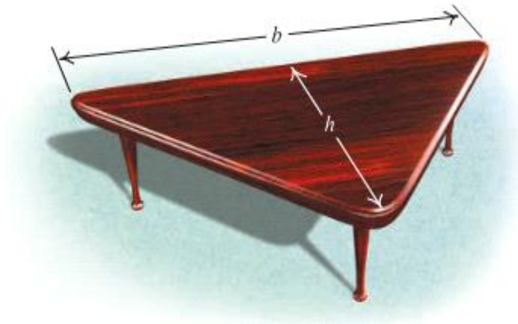

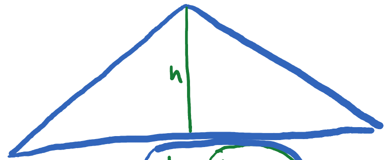

$b = h+20$

$A = \boxed{\frac{1}{2} b h}$

$A = \boxed{\frac{1}{2}(h+20)h = 750}$

$2 \cdot \frac{1}{2}(h+20)h = 750 \cdot 2$

$(h+20)h = 1500$

$h^2 + 20h = 1500$

$h^2 + 20h - 1500 = 0$

$(h+50)(h-30) = 0$

$h+50=0$ or $h-30=0$

$h = -50$ or $h = 30$ in

$b = 30+20$

$= 50$ in

So, the ht. is 30 in, and the base is 50 in.

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96. ***Safety flares.*** Suppose that a flare is launched upward with an initial velocity of 80 ft/sec from a height of 224 ft. Its height in feet, $h(t)$, after t seconds is given by

$$\boxed{h(t) = -16t^2 + 80t + 224.}$$

$h(2)$ = ht after 2 sec.

How long will it take the flare to reach the ground?

$\underbrace{\text{ground}}_{\text{ht. } 0}$

$0 = -16t^2 + 80t + 224$

$\frac{0}{-16} \quad \frac{-16t^2}{-16} \quad \frac{80t}{-16} \quad \frac{224}{-16}$

$$\frac{0}{-16} = \frac{-16t^2}{-16} + \frac{80t}{-16} + \frac{224}{-16}$$

$$0 = t^2 - 5t - 14$$

$$0 = (t+2)(t-7)$$

$t = 7$ sec