

Rational Functions

Goal: To simplify, multiply, and divide rational expressions and functions.

Definition: A rational expression is a ratio of two polynomials.

$$\textcircled{\text{ex}} \quad \frac{5}{6}, \quad \frac{x^2 + 7x + 10}{x - 8}$$

Note: $y = (\text{rational expression})$ is a function!

$$y = \frac{x^2 + 7x + 10}{x - 8} \quad \text{passes VLT}$$

$$\textcircled{\text{ex}} \quad \text{Let } f(x) = \frac{x^2 + 4x - 9}{x - 4}. \text{ Evaluate...}$$

$$\text{a) } \textcircled{f(0)}$$

$$= \frac{0^2 + 4(0) - 9}{0 - 4}$$

$$= \frac{-9}{-4}$$

$$= \textcircled{\frac{9}{4}}$$

$$\text{b) } f(-2)$$

$$f(-2) = \frac{(-2)^2 + 4(-2) - 9}{-2 - 4}$$

$$= \frac{4 - 8 - 9}{-6}$$

$$= \frac{-13}{-6}$$

$$= \frac{13}{6}$$

$$\text{c) } f(4)$$

undefined. So, 4 is not in the domain of f .

Dom f: $\{x \mid x \neq 4\}$

ex) Simplify the rational expression

a) $\frac{10yz^4}{40y^2z^9}$

$\frac{10}{40} \cdot \frac{y^1}{y^2} \cdot \frac{z^4}{z^9}$

$\frac{1}{4} \cdot \frac{y^1}{y^1} \cdot \frac{1}{z^5}$

$= \frac{1}{4yz^5}$

$\frac{z^4 \cdot 1}{z^4 \cdot z^5}$

$\frac{z^4}{z^9}$

$\frac{z^4}{z^4} \cdot \frac{1}{z^5}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

$\frac{10yz^4}{40y^2z^9}$

$\frac{1}{4yz^5}$

← work you need to show

b) $\frac{4y-20}{4y+12}$

factor everything!

$= \frac{4(y-5)}{4(y+3)}$

$= \frac{y-5}{y+3}$

ex) Simplify the rational function. List all domain restrictions: $h(t) = \frac{t^2 - 3t - 4}{t^2 + 9t + 8}$

$$h(t) = \frac{\cancel{(t+1)}(t-4)}{(t+8)\cancel{(t+1)}}, \quad (t \neq -8, t \neq -1)$$

$$h(t) = \frac{t-4}{t+8}, \quad \underbrace{t \neq -8, t \neq -1}_{\text{restrictions}}$$

ex) Multiply and Simplify

$$a) \frac{5t^3}{4t-8} \cdot \frac{6t-12}{10t}$$

$$\frac{\cancel{5}t^3}{\cancel{4}(t-2)} \cdot \frac{\cancel{6}(t-2)}{\cancel{2}10t}$$

$$\frac{3t^2}{4}$$

$$\frac{\cancel{3}}{\cancel{2}14} \cdot \frac{\cancel{21}^3}{23}$$

$$= \frac{9}{46}$$

$$\frac{3}{14} \cdot \frac{21}{23}$$

$$\frac{3}{\cancel{2}7} \cdot \frac{3 \cdot \cancel{7}}{23}$$

$$\left(\frac{7}{7}\right) \cdot \frac{3}{2} \cdot \frac{3}{23}$$

$$b) \frac{t^3-27}{t^4-9t^2} \cdot \frac{t^5-6t^4+9t^3}{t^2+3t+9}$$

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

$$\frac{(t-3)\cancel{(t^2+3t+9)}}{t^2(t^2-9)} \cdot \frac{t^3(t^2-6t+9)}{\cancel{(t^2+3t+9)}}$$

$$\frac{1 \cdot \cancel{(t-3)}}{\cancel{t}(\cancel{t-3})(t+3)} \cdot \frac{t^3(t-3)(t-3)}{1}$$

$$\cancel{x^2} (\cancel{x-3})(x+3) \quad (1)$$

$$= \frac{x(x-3)^2}{x+3}$$

2*) Divide and simplify. List all domain restrictions

$$f(x) = \frac{x^2 - 16}{x^2 - 10x + 25} \div \frac{3x - 12}{x^2 - 3x - 10}$$

$$f(x) = \frac{x^2 - 16}{x^2 - 10x + 25} \cdot \frac{x^2 - 3x - 10}{3x - 12}$$

$$= \frac{(x+4)(\cancel{x-4})}{(x-5)} \cdot \frac{(\cancel{x-5})(x+2)}{3(\cancel{x-4})}$$

$$= \frac{(x+4)(x+2)}{3(x-5)}, \quad x \neq 5, 4, -2$$

$$\frac{3}{4} \div \left(\frac{5}{2}\right)$$

$$\frac{3}{4} \cdot \frac{2}{5}$$