Complex Rational Expressions

Goal: To simplify fractions with fraction in the numerator and/or denominator.

(ex) Simplify

a) $$\dfrac{\left(\dfrac{4}{x^2} + \dfrac{3}{x^4}\right)}{\left(\dfrac{3}{x} - \dfrac{3}{x^3}\right)} \cdot \dfrac{x^4}{x^4}$$

$$\boxed{\dfrac{4x^2 + 3}{3x^3 - 3x}}$$

$$\dfrac{\dfrac{4x^2}{x^2} + \dfrac{3x^4}{x^4}}{\dfrac{3x^4}{x} - \dfrac{3x^4}{x^3}}$$

Method: Multiply numerator and denominator of the "big fraction" by the LCD of all the "little fractions" and simplify.

b) $$\dfrac{x^{-1} + y^{-1}}{\dfrac{x^2 - y^2}{xy}}$$

$$= \dfrac{\dfrac{1 \cdot xy}{x} + \dfrac{1 \cdot xy}{y}}{x^2 - y^2}$$

$$\dfrac{\left(\dfrac{1}{x} + \dfrac{1}{y}\right)}{\left[\dfrac{x^2 - y^2}{xy}\right]} \cdot \dfrac{xy}{xy}$$

$$= \dfrac{x + y}{x^2 - y^2}$$

$$= \dfrac{(x + y) \cdot 1}{(x + y)(x - y)}$$

$$= \boxed{\dfrac{1}{x-y}}$$

c)

$$\dfrac{\dfrac{x}{x^2+5x-6} + \dfrac{x}{x^2+5x-6}}{\dfrac{x}{x^2-5x+4} + \dfrac{2}{x^2-5x+4}}$$

factor every little DEN so you can find the LCD.

$$\dfrac{\left[\dfrac{x}{(x-1)(x+6)} + \dfrac{x}{(x-1)(x+6)}\right]}{\left[\dfrac{x}{(x-1)(x-4)} + \dfrac{2}{(x-1)(x-4)}\right]} \cdot \dfrac{(x-1)(x+6)(x-4)}{(x-1)(x+6)(x-4)}$$

$$\dfrac{\dfrac{x \,(x-1)(x+6)(x-4)}{(x-1)(x+6)} + \dfrac{x \,(x-1)(x+6)(x-4)}{(x-1)(x+6)}}{\dfrac{x \,(x-1)(x+6)(x-4)}{(x-1)(x-4)} + \dfrac{2\,(x-1)(x+6)(x-4)}{(x-1)(x-4)}}$$

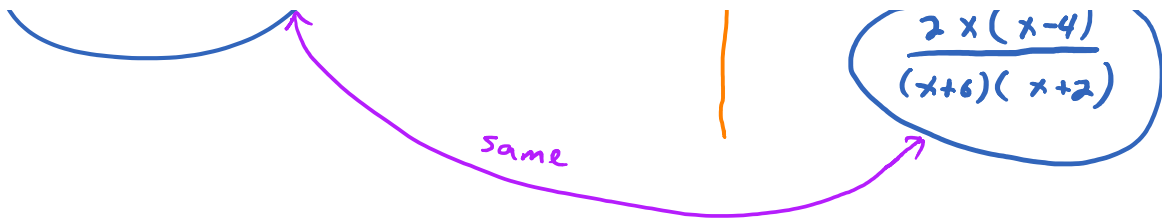$$\dfrac{1x\,(x-4) + 1x\,(x-4)}{x\,(x+6) + 2\,(x+6)}$$

$$\boxed{\dfrac{2x\,(x-4)}{(x+2)\,(x+6)}}$$

same

Easier to combine Num and DEN into single fractions and then flip and multiply:

$$\dfrac{\dfrac{2x}{x^2+5x-6}}{\dfrac{x+2}{x^2-5x+4}}$$

$$\dfrac{2x}{(x-1)(x+6)} \cdot \dfrac{(x-1)(x-4)}{x+2}$$

$$\boxed{\dfrac{2x\,(x-4)}{(x+6)\,(x+2)}}$$

$$\left(\frac{2x(x-4)}{(x+6)(x+2)} \right)$$

same

d)

$$\frac{\dfrac{1}{a^2-3a+2} + \dfrac{1}{a^2-4}}{\dfrac{1}{a^2+4a+4} + \dfrac{1}{a^2-4}}$$

$$\frac{\left[\dfrac{1}{(a-2)(a-1)} + \dfrac{1}{(a-2)(a+2)} \right]}{\left[\dfrac{1}{(a+2)^2} + \dfrac{1}{(a-2)(a+2)} \right]} \cdot \frac{(a-2)(a-1)(a+2)}{(a-2)(a-1)(a+2)}$$

$$\frac{(a+2)^2 + (a-1)(a+2)}{(a-2)(a-1) + (a-1)(a+2)}$$

$$\frac{\big((a+2) + (a-1) \big)(a+2)}{\big((a-2) + (a+2) \big)(a-1)}$$

$$\boxed{\dfrac{(2a+1)(a+2)}{2a(a-1)}}$$

ex Simplify :

$$\dfrac{\dfrac{x^2+7x+10}{x^2-x-12}}{\dfrac{x^2-5x-14}{x^2+7x+12}}$$

When you have single fractions in the big numerator and big denominator, simply flip and multiply the big numerator times the big denominator. Then factor and simplify as in section 6.1.

flip big DEN and multiply:

$$= \dfrac{x^2+7x+10}{x^2-x-12} \cdot \dfrac{x^2+7x+12}{x^2-5x-14}$$

Now simplify as you did in section 6.1

$$= \dfrac{(x+2)(x+5)}{(x+3)(x-4)} \cdot \dfrac{(x+3)(x+4)}{(x+2)(x-7)}$$

$$= \boxed{\dfrac{(x+5)(x+4)}{(x-4)(x-7)}}$$