Solving Formulas and Variation

Goal: To solve formulas for a specified variable and to solve variation applications.

Two Principles to Think About when Solving Eqns.

① The Reversibility Principle

Any op can be undone by applying reverse op (inverse op).

② The Balance Principle

Any op applied to one side must also be applied to the other side.

Ex Solve for the specified variable

a) $\boxed{\frac{W}{w} = \frac{D}{d}}$; W

$$\frac{W}{\cancel{w}} \cdot \cancel{w} = \frac{D}{d} \cdot \frac{w}{1}$$

$$W = \frac{Dw}{d}$$

b) $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$; R

$$\cancel{R} r_1 r_2 \cdot \frac{1}{\cancel{R}} = \left[\frac{1}{r_1} + \frac{1}{r_2}\right] R\, r_1 r_2$$

$$r_1 r_2 = \frac{R \cancel{r_1} r_2}{\cancel{r_1}} + \frac{R r_1 \cancel{r_2}}{\cancel{r_2}}$$

$$r_1 r_2 = R r_2 + R r_1$$

$$\frac{r_1 r_2}{r_2 + r_1} = \frac{R \cdot \cancel{(r_2 + r_1)}}{\cancel{(r_2 + r_1)}} \rightarrow \boxed{R_1 = \frac{r_1 r_2}{r_1 + r_2}}$$

c) $\frac{E}{e} = \frac{R+r}{R}$; R

$$ER = e(R+r)$$

$$ER = \cancel{eR} + er$$

$$-eR \qquad \cancel{-eR}$$

$$ER - eR = er$$

$$\frac{R \cdot \cancel{(E-e)}}{\cancel{E-e}} = \frac{er}{E-e} \rightarrow \boxed{R = \frac{er}{E-e}}$$

d) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$; a^2

$$a^2 b^2 \left(\frac{x^2}{b^2} + \frac{y^2}{a^2} \right) = 1 \cdot a^2 b^2$$

$$\cancel{a^2 x^2} + b^2 y^2 = a^2 b^2$$

$$\cancel{-a^2 x^2} \qquad -a^2 x^2$$

$$b^2 y^2 = a^2 b^2 - a^2 x^2$$

$$\frac{b^2 y^2}{b^2 - x^2} = \frac{a^2 \cancel{(b^2 - x^2)}}{\cancel{(b^2 - x^2)}}$$

$$a^2 = \frac{b^2y^2}{b^2 - x^2}$$

Guidelines for Solving a Rational Equation for a Specified Variable

Note that the following steps are only applied if necessary:

1. Clear the denominators by multiplying through by the LCD.
2. Distribute to get rid of parentheses.
3. Get every term with the specified variable on one side of the equal sign, everything else on the other side.
4. Factor out the specified variable.
5. Multiply or divide as needed in order to solve for the specified variable.

Variation

Three Key Phrases:

constant of variation

1. "y varies **directly** to x" means y = kx, where k is a constant.
2. "y varies **inversely** to x" means y = k/x, where k is a constant.
3. " z varies **jointly** to x and y" means z = kxy, where k is a constant.

ex Let z = 1600 and y = 80 when x = 5
Find the variation constant and the equation of variation when ...

a) y varies $\boxed{\text{directly}}$ as x.

$\boxed{y = kx}$

$80 = k5$

$\frac{80}{5} = k$

$k = 16$ ← constant of variation

$y = 16x$

eqn. of var.

b) y varies inversely as x

$y = \frac{k}{x}$

$80 = \frac{k}{5}$

$80 \cdot 5 = k$

$k = 400$

$y = \frac{400}{x}$

c) z varies jointly to x and y.

$z = kxy$

$\frac{1600}{80} = \frac{k \cdot 5 \cdot 80}{80}$

$20 = k \cdot 5$

$k = 4$

$z = 4xy$

62. *Weight on Mars.* The weight M of an object on Mars varies directly as its weight E on Earth. A person who weighs 95 lb on Earth weighs 38 lb on Mars. How much would a 100-lb person weigh on Mars?

p. 417

Plug in given measurements Solve for k → $\boxed{M = kE}$

$38 = k95$

$\boxed{M = \frac{2}{5}E}$ ← Plug in 100 to answer question.

$M = \frac{2}{5}(100)$

Solve for k

$38 = k95$

$k95 = 38$

$k = \frac{38}{95} = \frac{2}{5}$

question.

$m = \frac{2}{5}(100)$

$= 2 \cdot 20$

$= 40$ lbs

80. *Intensity of a signal.* The intensity I of a television signal varies inversely as the square of the distance d from the transmitter. If the intensity is 25 W/m^2 at a distance of 2 km, what is the intensity 6.25 km from the transmitter?

p. 418

$y = \frac{k}{x}$

$I = \frac{k}{d^2}$

$25 = \frac{k}{2^2}$

$\frac{k}{4} = 25$

$k = 100$

$I = \frac{100}{d^2}$

$I = \frac{100}{(6.25)^2}$ W/m^2