Radical Expressions and Functions

Goal: To simplify radical expressions

Def: The number c is a **square root of** a iff $c^2 = a$.

ex Find the square roots of...

a) 16 b) 144

4, since $4^2 = 16$ 12, -12

-4, since $(-4)^2 = 16$

Note: the radical symbol "$\sqrt{\ }$" refers to the principal (or positive) square root.

ex $\sqrt{16} = 4$, $\sqrt{144} = 12$

Def: The number c is an nth root of

Def: The number c is an nth root of a iff $c^n = a$. In symbols, $\sqrt[n]{a}$ denotes the principal nth root of a (n is called the index)

"nth root"

Ex Simplify

a) $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ since $2^3 = 8$

b) $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$ since $3^4 = 81$

c) $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$ since $(-2)^3 = -8$

d) $\sqrt[4]{(-2)^4} = \sqrt{16} = |-2| = 2$ since $(-2)^4 = 16$

Def: a) $\sqrt[n]{a^n} = |a|$ when n is even

b) $\sqrt[n]{a^n} = a$ when n is odd

Tips: ① $\sqrt{AB} = \sqrt{A} \cdot \sqrt{B}$

$\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$

② $\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$

③ $\sqrt{A \pm B} \neq \sqrt{A} \pm \sqrt{B}$

④ $|AB| = |A|\,|B|$

ex Simplify

a) $\sqrt{\frac{361}{16}}$

$= \frac{\sqrt{361}}{\sqrt{16}}$

$= \frac{19}{4}$

b) $-\sqrt{81}$

-9

c) $\sqrt{-81}$

undefined

d) $\sqrt{16x^2}$

$|4x|$

undefined

$c^2 = -81$ ← (no real c that satisfies)

$|4 \cdot x|$

$|4||x|$

$4|x|$

e) $\sqrt[3]{(7xy)^3}$

$7xy$

f) $\sqrt{(-7c)^2}$

$= |-7c|$

$= |-7|\,|c|$

$= 7|c|$

g) $\sqrt[4]{625 \cdot x^4}$

$\sqrt[4]{625}\ \sqrt[4]{x^4}$

$5|x|$

h) $\sqrt{9 - 6b + b^2}$

$\sqrt{(3-b)^2}$

$= |3-b|$

ex) Simplify. Assume that no radicands were made to take on negative quantities to even

(ex) Simplify. Assume [illegible] by taking negative quantities to even powers.

$x \geq 0$

a) $\sqrt{16x^2}$

$4x$

b) $\sqrt{b^2 - 6b + 9}$

$\sqrt{(b-3)^2}$

$= b - 3$

(ex) Consider $f(x) = \sqrt{x-1}$

a) find $f(-2)$ and $f(5)$

$f(-2) = \sqrt{(-2)-1} = \sqrt{-3}$ undefined

-2 is not in the domain of f.

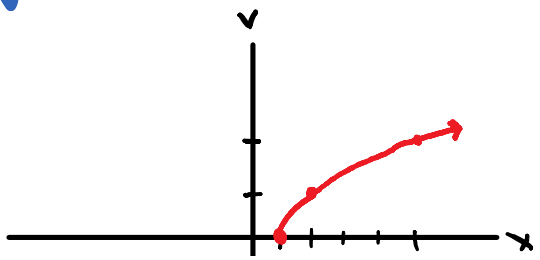
$f(5) = \sqrt{5-1} = \sqrt{4} = 2$

b) Graph $f(x) = \sqrt{x-1}$

x	$y = f(x) = \sqrt{x-1}$
1	0
2	1
5	2
10	3

$\sqrt{1-1} = \sqrt{0} = 0$

$\sqrt{2-1} = \sqrt{1} = 1$



D: $[1, \infty)$

R: $[0, \infty)$