

Rational Exponents

goal: To use simplify radical expressions using rational exponents.

Def: $a^{\frac{1}{n}} = \sqrt[n]{a}$ ($a \geq 0$ when n is even)

(ex) simplify

a) $49^{\frac{1}{2}}$

$$\sqrt{49}$$
$$\textcircled{7}$$

b) $-27^{\frac{1}{3}}$

$$-\sqrt[3]{27}$$
$$\textcircled{-3}$$

Def: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

(ex) simplify

a) $8^{\frac{4}{3}}$

$$= \sqrt[3]{8^4}$$

$$= (\sqrt[3]{8})^4$$

$$= 2^4 = \textcircled{16}$$

b) $81^{\frac{3}{4}}$

$$= (\sqrt[4]{81})^3$$

$$= (3)^3$$

$$= \textcircled{27}$$

Def: $a^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{a})^m}$ $a \neq 0$

Def: $a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \left(\frac{1}{a}\right)^{\frac{m}{n}}, a \neq 0$

(ex) simplify

a) $\frac{8^{-2}}{1}$

$\frac{1}{8^2}$

$\frac{1}{64}$

b) $\frac{1}{x^{-t}}$
 x^t

$\frac{1}{x^{-t}} = \frac{1}{\frac{1}{x^t}} = 1 \cdot \frac{x^t}{1} = x^t$

c) $\frac{3^{-\frac{5}{2}} a^3 b^{-\frac{7}{3}}}{1}$
 $\frac{a^3}{3^{\frac{5}{2}} \cdot b^{\frac{7}{3}}}$

d) $\left(\frac{2ab}{3c}\right)^{\frac{5}{6}}$
 $\left(\frac{3c}{2ab}\right)^{\frac{5}{6}}$

Laws of exponents

① $a^m a^n = a^{m+n}$
 $a^m a^{-n} = a^{m-n}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
 ④ $(ab)^m = a^m b^m$

$$\textcircled{2} \quad \frac{a^m}{a^n} = a^{m-n}$$

$$\textcircled{5} \quad a^1 = a$$

$$\rightarrow \textcircled{3} \quad (a^m)^n = a^{mn}$$

$$\textcircled{6} \quad a^0 = 1$$

These Laws hold for rational exponents!

$\textcircled{\text{ex}}$ simplify

$$\text{a) } 5^{\frac{2}{3}} \cdot 5^{\frac{1}{2}}$$

bases are 5

$$= 5^{\frac{2}{3} + \frac{1}{2}}$$

$$= \textcircled{5^{\frac{7}{6}}}$$

$$\left[\begin{array}{l} \frac{2}{3} \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{3}{3} \\ \frac{4}{6} + \frac{3}{6} \\ \frac{7}{6} \end{array} \right]$$

$$\text{b) } \frac{9^{\frac{7}{11}}}{9^{-\frac{2}{11}}} = 9^{\frac{7}{11}} \cdot 9^{\frac{2}{11}}$$

$$9^{\frac{7}{11} + (\frac{2}{11})}$$

$$\textcircled{9^{\frac{9}{11}}}$$

$$\text{c) } \left(a^{-\frac{1}{3}} \right)^{\frac{3}{2}}$$

$$= a^{-\frac{1}{3} \cdot \frac{3}{2}}$$

$$= a^{-\frac{1}{2}}$$

$$= \textcircled{\frac{1}{a^{\frac{1}{2}}}}$$

ex simplify using rational exponents

$$a) \sqrt[6]{x^4}$$

$$= x^{\frac{4}{6}}$$

$$= x^{\frac{2}{3}}$$

$$= \sqrt[3]{x^2}$$

$$b) \left(\sqrt[8]{(2x)}\right)^6$$

$$= (2x)^{\frac{6}{8}}$$

$$= (2x)^{\frac{3}{4}}$$

$$= \sqrt[4]{(2x)^3} = \left(\sqrt[4]{2x}\right)^3$$

$$= \sqrt[4]{2^3 x^3}$$

$$= \sqrt[4]{8x^3}$$

$$c) \left(\sqrt[5]{a^2 b^4}\right)^{15}$$

$$= (a^2 b^4)^{\frac{15}{5}}$$

$$= (a^2 b^4)^3$$

$$= (a^2)^3 (b^4)^3$$

$$= a^6 b^{12}$$