

More Fun with Radicals

Goal: to multiply and simplify radical expressions.

Big Rules: ① $\sqrt[n]{A} \sqrt[n]{B} = \sqrt[n]{AB}$
or
 $\sqrt[n]{AB} = \sqrt[n]{A} \sqrt[n]{B}$

$$\begin{array}{ccccccc} \sqrt{4} & \sqrt{9} & = & \sqrt{4 \cdot 9} & = & \sqrt{36} & = & 6 \\ 2 & \cdot & 3 & \xrightarrow{\hspace{10em}} & & & & \end{array}$$

② $\sqrt[n]{A^n} = A, \quad A \geq 0.$

②x multiply $\sqrt{2x} \cdot \sqrt{14y}$

$$= \sqrt{2x \cdot 14y}$$

$$= \sqrt{28xy}$$

Definition: A radical expression is **simplified** if no factors of the radicand can be written in terms of powers equal to or greater than the index.

$\sqrt{5^2}$, $\sqrt{5^3}$	← not simplified
$\sqrt{3}$, $\sqrt[4]{2^3}$	← simplified

ex Simplify

a) $\sqrt{32}$

$$\sqrt{2^5}$$

$$\sqrt{2^4 \cdot 2}$$

$$\sqrt{2^4} \cdot \sqrt{2}$$

$$2^{\frac{4}{2}} \sqrt{2}$$

$$2^2 \sqrt{2} = \boxed{4\sqrt{2}}$$



fast way

$$\sqrt{32}$$

$$= 4\sqrt{16 \cdot 2}$$

$$= \sqrt{16} \sqrt{2}$$

$$= 4\sqrt{2}$$

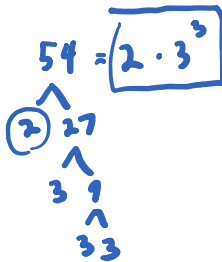
b) $\sqrt{54}$

$$\sqrt{2 \cdot 3^3}$$

$$3 \sqrt{2 \cdot 3}$$

$$\sqrt{3^2} \cdot \sqrt{6}$$

$$3\sqrt{6}$$



c) $\sqrt[3]{48}$

$$\sqrt[3]{2^4 \cdot 3}$$

$$\sqrt[3]{2^3 \cdot 2 \cdot 3}$$

$$\sqrt[3]{2^3} \sqrt[3]{6}$$

ex Perfect 3rd powers of x

48 = 2⁴ · 3

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  48
  /
 16
 /
 4
 /
 4
 /
 2
 /
 2
  
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$$x^3$$

$$x^6$$

$$x^9$$

$$x^{12} = (x^4)^3$$

$$\sqrt[3]{x^{12}} = x^{\frac{12}{3}} = x^4$$

$$\sqrt[3]{2^3} \sqrt[3]{6}$$

$$\textcircled{2\sqrt[3]{6}}$$

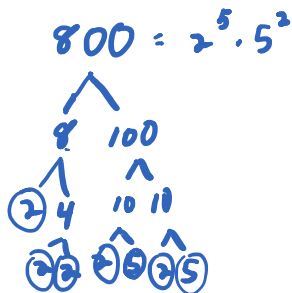
$$\sqrt[3]{x^{12}} = x^{\frac{12}{3}} = x^4$$

d) $\sqrt[4]{800}$

$$\sqrt[4]{2^5 \cdot 5^2}$$

$$2\sqrt[4]{\cancel{2^4} \cdot 2 \cdot 5^2}$$

$$\textcircled{2\sqrt[4]{50}}$$



ex) Simplify (assume variables ≥ 0)

a) $\sqrt{x^5}$

$$x^2 \sqrt{\cancel{x^4} \cdot x}$$

$$\textcircled{x^2 \sqrt{x}}$$

b) $\sqrt[3]{x^{14}}$

$$\sqrt[3]{x^{12} \cdot x^2}$$

$$\sqrt[3]{x^{12}} \sqrt[3]{x^2}$$

$$\textcircled{x^4 \sqrt[3]{x^2}}$$

c) $\sqrt[4]{y^{17}}$

$$y^6 \sqrt[4]{y^{24} y^3}$$

$$y^6 \sqrt[4]{y^3}$$

$$d) \sqrt[3]{72 a^4 b^3 c^{10}}$$

$$2 \cdot 3^2 b^3 \sqrt[3]{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{6} \cdot (2b)}$$

$$6 a^2 b c^5 \sqrt[3]{2b}$$

$$72 = 2^3 \cdot 3^2$$

$$\begin{array}{c} \wedge \\ 8 \quad 9 \\ \downarrow \quad \downarrow \\ 2^3 \quad 3^2 \end{array}$$

ex) multiply and simplify

$$\sqrt{5a^7} \cdot \sqrt{15a^4}$$

$$\sqrt{5a^7 \cdot 15a^4}$$

$$5a^5 \sqrt{\cancel{15} \cdot \cancel{15}} (3a)$$

$$5a^5 \sqrt{3a}$$

$$5 \cdot 15$$

$$5 \cdot 5 \cdot 3$$

$$5^2 \cdot 3$$

$$a^{11}$$

$$a^{10} \cdot a$$

You Try It!

$$\sqrt[3]{24 x^7 y^4}$$

$$2x^2 y \sqrt[3]{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot (3xy)}$$

$$24 = 2^3 \cdot 3$$

$$\begin{array}{c} \wedge \\ 3 \quad 8 \end{array}$$

$$2x^2y\sqrt[3]{3xy}$$