

The Four Operations on Radicals

Goal: to add, subtract, multiply, and divide radicals.

Ⓧ Add or subtract

a) $8\sqrt{5} + 6\sqrt{5}$

$14\sqrt{5}$

b) $8\sqrt{3} + 3\sqrt[3]{3}$

can't combine (unlike indices)

c) $2\sqrt[3]{81} - 4\sqrt[3]{3}$

simplify $\sqrt{\quad}$ s first!

$2 \cdot 3 \sqrt[3]{3 \cdot 3} - 4\sqrt[3]{3}$

$6\sqrt[3]{3} - 4\sqrt[3]{3}$

$2\sqrt[3]{3}$

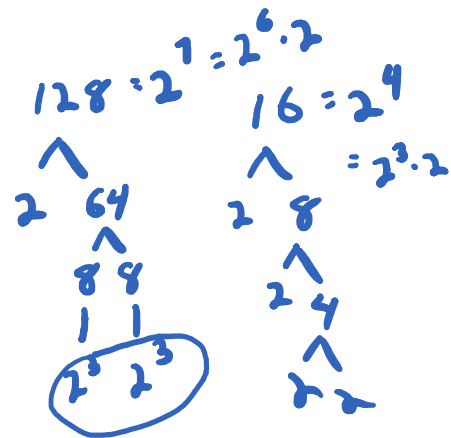
$81 = 3^4 = 3^3 \cdot 3$
 $\begin{array}{c} \wedge \\ 9 \ 9 \\ \wedge \ \wedge \\ 3 \ 3 \ 3 \ 3 \end{array}$

$$d) 2b\sqrt[3]{16b^2} + 4\sqrt[3]{128b^5}$$

$$2 \cdot 2b \sqrt[3]{2^3 \cdot 2b^2} + 4 \cdot 4b \sqrt[3]{2^3 \cdot 2^3 \cdot 2b^2}$$

$$4b\sqrt[3]{2b^2} + 16b\sqrt[3]{2b^2}$$

$$20b\sqrt[3]{2b^2}$$



$$e) \sqrt{9x-9} - \sqrt{x^3-x^2}$$

$$3\sqrt{9(x-1)} - 1x\sqrt{x^2(x-1)}$$

$$3\sqrt{x-1} - 1x\sqrt{x-1}$$

$$(3 - |x|)\sqrt{x-1}$$

ex multiply

$$a) (3\sqrt{5})(2\sqrt{7})$$

$$3 \cdot 2 \cdot \sqrt{5} \sqrt{7}$$

$$6\sqrt{35}$$

$$b) 4\sqrt[3]{5} (2\sqrt[3]{25} - 2)$$

$$4\sqrt[3]{5} \cdot 2\sqrt[3]{25} - 4\sqrt[3]{5} \cdot 2$$
$$8\sqrt[3]{5^3} - 8\sqrt[3]{5}$$

$$8 \cdot 5 - 8\sqrt[3]{5}$$

$$\rightarrow 40 - 8\sqrt[3]{5}$$

$$c) (3\sqrt{x} + 4)(\sqrt{x} + 2)$$

(assume $x \geq 0$)

$$3\sqrt{x}\sqrt{x} + 3\sqrt{x} \cdot 2 + 4\sqrt{x} + 8$$

$$3x + 6\sqrt{x} + 4\sqrt{x} + 8$$

$$3x + 10\sqrt{x} + 8$$

$$b^{\frac{4}{3}} b^{\frac{3}{4}}$$

$$d) \sqrt[3]{b^4} \sqrt[4]{b^3}$$

$$(b^4)^{\frac{1}{3}} (b^3)^{\frac{1}{4}}$$

$$(b^4)^{\frac{4}{12}} (b^3)^{\frac{3}{12}}$$

$$[(b^4)^4 (b^3)^3]^{\frac{1}{12}}$$

$$[b^{16} b^9]^{\frac{1}{12}}$$

$$\sqrt[12]{b^{25}}$$

$$b^2 \sqrt[12]{b^4 b}$$

$$b^2 \sqrt[12]{b^5}$$

$$e) \sqrt{2x^3y^3} \sqrt[3]{4xy^2}$$

$$(2x^3y^3)^{\frac{1}{2}} (4xy^2)^{\frac{1}{3}}$$

$$(2x^3y^3)^{\frac{2}{6}} (4xy^2)^{\frac{2}{6}}$$

$$\sqrt[6]{(2x^3y^3)^2 (4xy^2)^2}$$

$$(ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$$

$$(a^m)^n = a^{mn}$$

$$\frac{1}{3} \cdot \frac{4}{4} = \frac{4}{12}$$

$$\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$$

$$b^{25} = b^{24} b$$

$$6$$

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\begin{aligned}
 & (2x^3y^2)^6 (4xy^2)^6 \\
 & [(2x^3y^2)^3 (4xy^2)^3]^{\frac{1}{2}} \\
 & [2^3(x^3)^3(y^2)^3 \cdot 4^2 x^2 (y^2)^2]^{\frac{1}{2}} \\
 & [8x^9y^6 \cdot 16x^2y^4]^{\frac{1}{2}} \\
 & \sqrt[6]{\cancel{2^6} \cancel{y^6}} (2x^5y) \\
 & 2x^5y \sqrt[6]{2x^5y}
 \end{aligned}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\begin{aligned}
 2^7 &= 2^6 \cdot 2 \\
 x^8 &= x^6 \cdot x^2 \\
 y^8 &= y^6 \cdot y^2
 \end{aligned}$$

Def: The conjugate of $a+b$ is $a-b$ and vice-versa.

$$(a+b)(a-b) = a^2 - b^2$$

(ex) Divide

$$a) \frac{7}{\sqrt{5} + 2}$$

multiply NUM and DEN by conj. of DEN



$$\frac{7}{(\sqrt{5} + 2)} \cdot \frac{(\sqrt{5} - 2)}{(\sqrt{5} - 2)}$$

$$\frac{7\sqrt{5} - 14}{5 - 4} = 7\sqrt{5} - 14$$

b)

$$\frac{(x - 4)(\sqrt{x} - \sqrt{y})}{(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})}$$

$$\frac{x\sqrt{x} - x\sqrt{y} - 4\sqrt{x} + 4\sqrt{y}}{x - y}$$