Quadratic Equations

Goal: to solve quadratic equations by taking roots and completing the square.

↳ $ax^2+bx+c=0$, where a, b, and c are constants.

(ex) Solve by taking roots

a) $(x)^2 = 16$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

$x = -4$ or $x = 4$

$\sqrt{x^2} = |x|$

$(-4)^2 = 16$ ✓

b) $x^2 + 8 = 0$

$-8 \quad -8$

$\sqrt{x^2} = \pm\sqrt{-8}$

$x = \pm i\sqrt{8}$

$x = \pm i\, 2\sqrt{2}$

$x = \pm 2i\sqrt{2}$

isolate x^2

$\sqrt{8}$

$2\sqrt{4\cdot 2}$

c) $(3y+7)^2 = 1$

$\sqrt{(3y+7)^2} = \pm\sqrt{1}$

$3y+7 = \pm 1$

$3y+7 = -1$ or $3y+7 = 1$

$3y = -8$ $\qquad$ $3y = -6$

$3y+1=-1$ or $3y+1=1$

$3y=-8$ $\quad$ $3y=-6$

$y=\frac{-8}{3}$ or $y=-2$

d) $x^2-12x+36=-8$

$\sqrt{(x-6)^2}=\pm\sqrt{-8}$

$x-6=\pm 2i\sqrt{2}$

$+6 \quad +6$

$x=6\pm 2i\sqrt{2}$

$x^2-12x+44=0$

$(x \quad)(x \quad)=0$

doesn't factor

ex Solve by completing the square

a) $x^2+10x+22=0$

$-22 \quad -22$

$1x^2+10x+25=-22+25$

$(x+5)^2=-22+25$

$\sqrt{(x+5)^2}=\pm\sqrt{3}$

$x+5=\pm\sqrt{3}$

$-5 \quad -5$

$x=-5\pm\sqrt{3}$ exact

$x\approx -6.73$ or $x\approx -3.27$ estimates

Recipe (one-variable)

1. Isolate constant
2. Divide through by the lead coefficient
3. Take $\frac{1}{2}$ of the coefficient of x. Square it, and add to both sides.
4. solve by taking roots

b) $5y^2+12y-1=0$

$+1 \quad +1$

$\frac{5y^2}{5}+\frac{12y}{5}=\frac{1}{5}$

$y^2+\frac{12}{5}y+\frac{36}{25}=\frac{5}{5}\cdot\frac{1}{5}+\frac{36}{25}$

$\frac{1}{2}\cdot\frac{6}{5}=\frac{6}{5}$

$\left(\frac{6}{5}\right)^2=\frac{36}{25}$

$y^2 + \frac{12}{5}y + \frac{36}{25} = \frac{5}{5}\cdot\frac{1}{5} + \frac{36}{25}$ | (5) (25)

$\left(y + \frac{6}{5}\right)^2 = \frac{5}{25} + \frac{36}{25}$

$\sqrt{\left(y + \frac{6}{5}\right)^2} = \pm\sqrt{\frac{41}{25}}$

$y + \frac{6}{5} = \pm\frac{\sqrt{41}}{5}$

$-\frac{6}{5}\quad -\frac{6}{5}$

$y = \frac{-6}{5} \pm \frac{\sqrt{41}}{5} = \frac{-6 \pm \sqrt{41}}{5}$

ex Solve for x: $ax^2 + bx + c = 0$, where a, b, and c are constants.

complete the square...

$\frac{ax^2}{a} + \frac{bx}{a} = \frac{-c}{a}$ | $\frac{1}{2}\cdot\frac{b}{a}$ $\left(\frac{b}{2a}\right)^2$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{4a}{4a}\left(-\frac{c}{a}\right) + \frac{b^2}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$ | $\frac{b^2}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

$-\frac{b}{2a}\quad -\frac{b}{2a}$

$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ → Quadratic Formula