

Examining the Solutions to Quadratic Equations

Goals:

1. To find the kinds of solutions that quadratic equations yield.
2. To build quadratic equations from given solutions.

Definition: The discriminant is the radicand from the quadratic formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant

$D = b^2 - 4ac$ Value of Discriminant	Nature of Solutions to $ax^2 + bx + c = 0$
<ol style="list-style-type: none"> ① $D < 0$ ② $D = 0$ ③ $D > 0$ 	<ol style="list-style-type: none"> ① Two complex solutions ^(a+bi) ✓ ② One real solution (rational) ③ Two real solutions <ol style="list-style-type: none"> a) If D is a perfect \square then 2 rational solutions b) If D is <u>not</u> a perfect square, then 2 irrational

b) If D is not a perfect square, then 2 irrational solutions

ex) Describe the solutions to the given equation. Classify each solution(s) according to number type.

a) $x^2 + 2x - 15 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 4 - 4(1)(-15) \\ &= 4 + 60 \\ &= 64 > 0 \end{aligned}$$

2 rational solutions

b) $3x^2 - 8x = 2$
 $3x^2 - 8x - 2 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 64 - 4(3)(-2) \\ &= 64 + 24 \\ &= 88 > 0 \end{aligned}$$

2 irrational solutions

c) $5x^2 + 7x + 3 = 0$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 49 - 4(5)(3) \end{aligned}$$

$$= 49 - 60$$

$$= -11 < 0$$

2 complex solutions

ex) Find a quadratic equation with the given solutions. Write your answer in the form $ax^2 + bx + c = 0$

a) 7, 4

$$(x-7)(x-4) = 0$$

$$x^2 - 11x + 28 = 0$$

$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$
$$x = 3 \text{ or } x = 4$$

b) $-\sqrt{2}, \sqrt{2}$

$$(x - (-\sqrt{2}))(x - \sqrt{2}) = 0$$

$$(x + \sqrt{2})(x - \sqrt{2}) = 0$$

$$x^2 - 2 = 0$$

c) $2 - 7i, 2 + 7i$

$$(x - (2 - 7i))(x - (2 + 7i)) = 0$$

$$(x - (2 - 7i))(x - (2 + 7i)) = 0$$

$$x^2 - x(2 + 7i) - x(2 - 7i) + (2 - 7i)(2 + 7i) = 0$$

$$x^2 - 2x - 7ix - 2x + 7ix + 4 - 49i^2 = 0$$

$$x^2 - 4x + 4 + 49 = 0$$

$$x^2 - 4x + 53 = 0$$