Applications of Quadratic Equations

Goals:

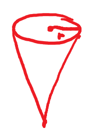
1. To solve formulas for a specified unknown.
2. To solve applications involving quadratic equations.

ex) Solve for the specified variable. Assume all variables are nonnegative.

a) $V = \frac{1}{3}\pi r^2 h$, for r

$$\frac{V}{\frac{1}{3}\pi h} = \frac{\cancel{\frac{1}{3}}\cancel{\pi} r^2 \cancel{h}}{\cancel{\frac{1}{3}}\cancel{\pi}\cancel{h}}$$

no r' in eqn. solve by taking roots.



$$\frac{3V}{\pi h} = r^2$$

$$\sqrt{r^2} = \sqrt{\frac{3V}{\pi h}}$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

b) $A = P(r^2+2r+1)$, for r

$$\frac{A}{P} = \frac{P(r^2+2r+1)}{P}$$

When you see r' (and r^2) in eqn, use Q.F. to solve.

$$\frac{A}{P} = r^2+2r+1 \quad -\frac{A}{P}$$

$$-\frac{A}{P}$$

$$0 = 1r^2+2r+\left(1-\frac{A}{P}\right)$$

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Another method

$$A = P(r+1)^2$$

$$\sqrt{\frac{A}{P}} = \sqrt{(r+1)^2}$$

$$\sqrt{\frac{A}{P}} = r+1$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-2 \pm \sqrt{4 - 4\left(1 - \frac{A}{P}\right)}}{2}$$

$$r = \frac{-2 \pm \sqrt{4 - 4 + \frac{4A}{P}}}{2}$$

$$r = \frac{-2 \pm \sqrt{\frac{4A}{P}}}{2}$$

same?

$$r = \frac{-2}{2} + \frac{\cancel{2}\sqrt{\frac{A}{P}}}{\cancel{2}}$$

$$r = -1 + \sqrt{\frac{A}{P}}$$

same

$$\sqrt{\frac{A}{P}} = r + 1$$

$$-1 \qquad -1$$

$$\sqrt{\frac{A}{P}} - 1 = r$$

$$r = \sqrt{\frac{A}{P}} - 1$$

c) $E = 4\sqrt{x} + e$, for x

$$-e \qquad\qquad -e$$

$$\frac{E - e}{4} = \frac{4\sqrt{x}}{4}$$

$$\left(\frac{E-e}{4}\right)^2 = \left(\sqrt{x}\right)^2$$

$$x = \left(\frac{E-e}{4}\right)^2$$

d) $V = rv^2 - 2hdv$, for v

$$-V \qquad\qquad -V$$

$$0 = rv^2 - (2hd)v - V$$

$a = r$, $b = -2hd$, $c = -V$

$$v = \frac{2hd \pm \sqrt{(-2hd)^2 - 4r(-V)}}{2r}$$

$$v = \frac{2hd \pm \sqrt{4h^2d^2 + 4rV}}{2r}$$

v^1, v^2 yes

So use Q.F.

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Guidelines for Solving a formula for a Specified Variable

Note that the following steps are only applied if necessary:

1. Clear the denominators and radicals and combine like terms.
2. Distribute to get rid of parentheses.
3. If the equation is linear with respect to the specified variable, then solve using previous methods. (no variable^2)
4. If the equation is quadratic with respect to the specified variable, then solve either by taking roots or the quadratic formula.

only variable^2 → (variable)^1 and (variable)^2

Ex. The height of a building is 566 meters. How long would it take an object to hit the ground if it were thrown from the top of the building with an initial velocity of 5 meters/sec?

$S = 4.9t^2 + V_0 t$

dist. time initial velocity.

$566 = 4.9t^2 + 5t$

$0 = 4.9t^2 + 5t - 566$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$t = \frac{-5 \pm \sqrt{25-4(4.9)(-566)}}{2(4.9)}$

$t = \frac{-5 + \sqrt{11118.6}}{9.8}$

$t \approx 10.2$ secs. ✓

Ex. Don's Dodge Durango travels 1085 miles. If he had gone 8 mph faster, then the trip would have taken 2 hours less time. Find the faster, then the trip would have taken 2 hours less time. Find the average speed of the Durango.

	D	= R	· T
Actual info	1085	r	$t = \frac{1085}{r}$
"what if"	1085	r+8	$t-2 = \frac{1085}{r+8}$

$\boxed{1085 = r \cdot t}$

$D = R \cdot T \rightarrow T = \frac{D}{R}$

$\rightarrow t = \frac{1085}{r+8} + 2$

$$\left[\frac{1085}{r+8} + 2 = \frac{1085}{r}\right] r(r+8)$$

$$1085r + 2r(r+8) = 1085(r+8)$$

$$\cancel{1085r} + 2r^2 + 16r = \cancel{1085r} + 8680$$

$$\frac{2r^2}{2} + \frac{16r}{2} - \frac{8680}{2} = \frac{0}{2}$$

$$1r^2 + 8r - 4340 = 0$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-8 \pm \sqrt{64 - 4(-4340)}}{2}$$

$$r = \frac{-8 \pm 132}{2}$$

$$r = \frac{-8 + 132}{2}$$

$$r = \frac{124}{2} = 62 \text{ mph}$$

Ex. Two different pipes can fill a swimming pool. When turned on at the same time, they fill pool in 8.4 hours. The smaller pipe alone takes 7 hours longer than larger pipe to fill the pool. How long would it take the larger pipe to fill the pool alone?

t = hrs takes larger pipe to fill pool.

$t + 7$ = " " smaller " " " ".

$\frac{1}{8.4}$ is amount of job finished in 1 hr. (together)

$\frac{1}{t}$ " " " " " " " " by larger pipe

$\frac{1}{t+7}$ " " " " " " " " by smaller pipe

$\frac{1}{\quad} + \frac{1}{\quad} = \frac{1}{8.4}$

$t+7$

$$\frac{1}{t}+\frac{1}{t+7}=\frac{1}{8.4}$$

$$8.4=8\tfrac{4}{10}$$

$$8\tfrac{2}{5}$$

$$\frac{42}{5}$$

$$\frac{1}{8.4}=\frac{5}{42}$$

$$42t(t+7)\left[\frac{1}{t}+\frac{1}{t+7}=\frac{5}{42}\right]$$

$$42(t+7)+42t=5t(t+7)$$

$$42t+294+42t=5t^2+35t$$

$$84t+294=5t^2+35t$$

$$-84t\quad -294 \qquad\qquad -84t\quad -294$$

$$0=5t^2-49t-294$$

$$t=\frac{49\pm\sqrt{(-49)^2-4(5)(-294)}}{10}$$

$t=-4.2$ or $t=14$ hrs.

So, it takes the larger pipe 14 hrs. to finish.