

# Quadratic-Like Equations

Goal: To solve equations that can be thought of as quadratic.

**ex** Solve

$$a) \underbrace{(y-3)}_u^2 - 5\underbrace{(y-3)}_u + 6 = 0$$

$$\text{let } \boxed{u = y-3} \text{ (make sub)}$$

$$u^2 - 5u + 6 = 0$$

$$(u - 2)(u - 3) = 0$$

$$u - 2 = 0 \quad \text{or} \quad u - 3 = 0$$

$$u = 2 \quad \text{or} \quad u = 3$$

$$\downarrow$$
$$y - 3 = 2$$

$$\downarrow$$
$$y - 3 = 3$$

← back sub

$$\boxed{y = 5 \quad \text{or} \quad y = 6}$$

$$b) \underbrace{x^4}_{u^2} + 3\underbrace{x^2}_u - 28 = 0$$

$$\text{let } \boxed{u = x^2}$$

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$$\text{let } (u = x^2)$$

$$\square \left\{ \begin{array}{l} \square \\ \rightarrow u^2 = (x^2)^2 = x^4 \end{array} \right.$$

$$|u^2 + 3u - 28 = 0$$

$$(u + 7)(u - 4) = 0$$

$$u + 7 = 0, \quad u - 4 = 0$$

$$u = -7 \quad \text{or} \quad u = 4$$

$$\downarrow \leftarrow$$
$$\sqrt{x^2} = \pm \sqrt{-7}$$

$$\downarrow \leftarrow$$
$$\sqrt{x^2} = \pm \sqrt{4}$$

Back sub

$$x = \pm \sqrt{-7}$$

$$x = \pm i\sqrt{7}$$

$$x = \pm 2$$

$$c) \quad 2m + \sqrt{m} = 15$$

$$2m^{\textcircled{1}} + m^{\textcircled{\frac{1}{2}}} = 15$$

$$u = m^{\frac{1}{2}}$$

$$u^2 = (m^{\frac{1}{2}})^2 = m$$

$$2u^2 + u = 15$$

$$\textcircled{2}u^2 + u - 15 = 0$$

$$(2u - 5)(u + 3) = 0$$

$$2u - 5 = 0 \quad \text{or} \quad u + 3 = 0$$

$$u = \frac{5}{2} \quad \text{or} \quad u = -3$$

$$\downarrow$$
$$\left(m^{\frac{1}{2}}\right)^2 = \left(\frac{5}{2}\right)^2$$

$$\left(m = \frac{25}{4}\right)$$

$$\downarrow$$
$$m^{\frac{1}{2}} = -3$$

$$\sqrt{m} = -3$$

wouldn't check  
(extraneous)

$$m = \frac{25}{4}$$

$$\begin{array}{l} \cancel{m = -3} \\ \cancel{m = 9} \end{array} \leftarrow \text{wouldn't be extraneous}$$

(ex) Find the x and y intercepts:

$$f(x) = x - 4\sqrt{x} - 1$$

y-int:

$$f(0) = 0 - 4\sqrt{0} - 1 = -1$$

$$(0, -1) \leftarrow \text{y-int.}$$

x-int:

$$0 = x - 4\sqrt{x} - 1$$

$$u = \sqrt{x}$$

$$u^2 = x$$

$$u^2 - 4u - 1 = 0$$

$$u = \frac{4 \pm \sqrt{16 - 4(-1)}}{2}$$

$$u = \frac{4 \pm \sqrt{20}}{2}$$

$$u = \frac{4 \pm 2\sqrt{5}}{2}$$

$$u = \frac{2 \pm \sqrt{5}}{1}$$

$$u = 2 - \sqrt{5} \text{ or } u = 2 + \sqrt{5}$$

$$\sqrt{x} = 2 - \sqrt{5} < 0 \text{ or } (\sqrt{x})^2 = (2 + \sqrt{5})^2$$

$$x = 4 + 4\sqrt{5} + 5$$

$$x = 9 + 4\sqrt{5}$$

$$\text{x-int.} \rightarrow (9 + 4\sqrt{5}, 0)$$