Applications of Quadratic Functions

Goals:

1. To find an extreme value of a quadratic function in an applied context.
2. To find the equation of a quadratic function that fits given data points.

Vertex Formula

$f(x) = ax^2 + bx + c$

⋮ complete ☐

$f(x) = a(x-h)^2 + k$

$h = \frac{-b}{2a}$, $k = f\left(\frac{-b}{2a}\right)$

$V(h, k)$

Ex. The profit for producing *x* Snickers bars (mmmm Snickers) is $P(x) = -2x^2 + 1720x - 369585$, where *P*(*x*) is in dollars. Use the vertex formula to find the number of Snickers bars needed to be produced to make a maximum profit and find the value of that maximum profit.

$h = \frac{-b}{2a} = \frac{-1720}{2(-2)} = 430$ snickers bars

(h: input)

max profit = $k = P(430) = -2(430)^2 + 1720(430) - 369585$

$= \$215.$

Ex. Suppose the cost, in dollars, of manufacturing *n* hundred widgets is given by $C(n) = 3n^2 - 2n + 11$. Find the number of widgets that minimizes cost and calculate the minimum cost.

$h = \frac{-b}{2a} = \frac{x}{x(3)} = \frac{1}{3}$ hundred widgets $= 33\frac{1}{3}$

≈ 33 widgets

$k = \text{min cost} = C\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) + 11$

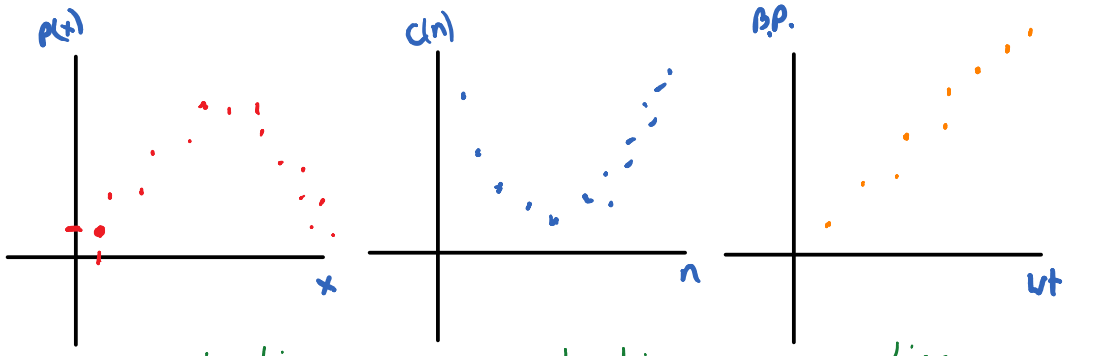
$= \frac{3}{9} - \frac{2}{3} + \frac{11}{1}\cdot\frac{3}{3}$

$= \frac{1}{3} - \frac{2}{3} + \frac{33}{3}$

$= -\frac{1}{3} + \frac{33}{3} = \frac{32}{3} = 10\frac{2}{3}$

$= \$10.67$

Def: A **scatter plot** is a graph of two-variable data points.



quadratic relationship between x and y

quadratic relationship

linear relationship

ex Find a quadratic function that fits the given points: $(1, 4), (-2, 16), (-1, 6)$

$f(x) = ax^2 + bx + c$

$f(x) = ax^2 + bx + c$

$ax^2 + bx + c = y$

$a(1)^2 + b(1) + c = 4 \rightarrow a + b + c = 4$

$a(-2)^2 + b(-2) + c = 16 \rightarrow 4a - 2b + c = 16$

$a(-1)^2 + b(-1) + c = 6 \rightarrow a - b + c = 6$

① $a + b + c = 4$

② $4a - 2b + c = 16$

③ $a - b + c = 6$

① + ③

$2a + 2c = 10$

$\div 2$

$a + c = 5$

2① + ②

$2a + 2b + 2c = 8$

$4a - 2b + c = 16$

$6a + 3c = 24$

$\div 3$

$2a + c = 8$

$2a + c = 8$

$-a + -c = -5$

$a = 3$

$3 + c = 5$

$c = 2$

$3 + b + 2 = 4$

$b + 5 = 4$

$b = -1$

$f(x) = ax^2 + bx + c$

$f(x) = 3x^2 - x + 2$