Polynomial and Rational Inequalities

Goal: To solve these things!

(ex) Solve

a) $3x^2 - 4x \leq 4$ (use same method as for solving abs. value inequalities)

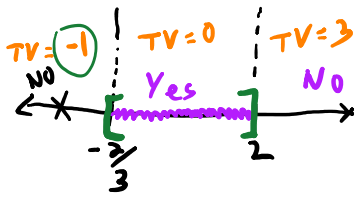
$3x^2 - 4x - 4 \leq 0$ include end points

$(3x+2)(x-2) \leq 0$

$(3x+2)(x-2) = 0$ } ① solve related eqn. to get critical values

$3x+2=0$ or $x-2=0$

$x = \frac{-2}{3}$ or $x = 2$ critical values



$(3x+2)(x-2) \leq 0$ ② Graph c.v.s on a number line

TV = -1

$(-3+2)((-1)-2)$

$(-)(-) \overset{?}{\leq} 0$

$(+) \leq 0$

No way!

TV = 0

$-4 \overset{?}{\leq} 0$

Yes!

TV = 3

$(+)(+) \overset{?}{\leq} 0$

No way!

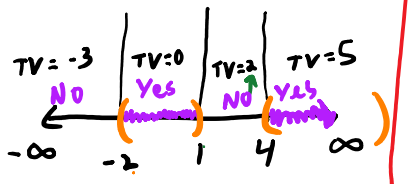
③ use test values to determine interval(s) of solution.

$\left[\frac{-2}{3}, 2\right]$

b) $(x-1)(x+2)(x-4) > 0$

$(x-1)(x+2)(x-4) = 0$

$x = 1, -2, 4$



TV = -3

$(-)(-)(-) \overset{?}{>} 0$

NO

TV = 0

$(-)(+)(-) \overset{?}{>} 0$

Yes!

$(x-1)(x+2)(x-4) > 0$

$(x-1)(x+2)(x-4) > 0$

c.v.s: $x=1, x=-2, x=4$

TV = 2: $(+)(+)(-) \overset{?}{>} 0$ — No

TV = 5: $(+)(+)(+) > 0$ — Yes

$(-2,1) \cup (4,\infty)$

ex Solve

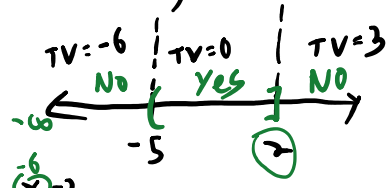
a) $\dfrac{x-2}{x+5} \leq 0$

$R(x)$

Get c.v.s by setting NUM = 0 and DEN = 0.

$x-2=0$, $x+5=0$

$x=2$, $x=-5$



$\dfrac{-6-2}{-6+5} \leq 0$

TV = -6: $\dfrac{(-)}{(-)} \overset{?}{\leq} 0$ NO!

TV = 0: $\dfrac{(-)}{(+)} \overset{?}{\leq} 0$ Yes!

TV = 3: $\dfrac{(+)}{(+)} \not\leq 0$

$(-5, 2]$

not included (make division by 0)

included

b) $\dfrac{y-5}{y^2+2y-8} < 0$ — don't include end points

$\dfrac{y-5}{(y-2)(y+4)} < 0$

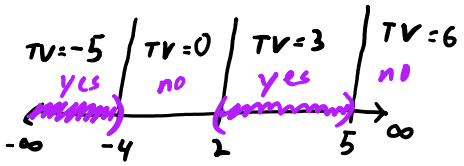
$y-5$ } set NUM = 0, DEN = 0

$(y-2)(y+4)$

$\frac{y-5}{(y-2)(y+4)} = 0$ } set NUM = 0, DEN = 0 to find critical values

$y-5=0$, $(y-2)(y+4)=0$

$y=5$, $y=2$, $y=-4$ C.V.s



$\frac{y-5}{(y-2)(y+4)} < 0$

TV = -5: $\frac{(-)}{(-)(-)} \overset{?}{<} 0$ Yes

TV = 0: $\frac{(-)}{(-)(+)} \overset{?}{<} 0$ NO!

TV = 3: $\frac{(-)}{(+)(+)} \overset{?}{<} 0$ Yes!

TV = 6: $\frac{+}{(+)(+)} \overset{?}{<} 0$ No

$(-\infty, -4) \cup (2, 5)$

c) $\frac{x}{x-1} > 2$

$\frac{x}{x-1} - 2\frac{(x-1)}{(x-1)} > 0$

$\frac{x-2(x-1)}{x-1} > 0$

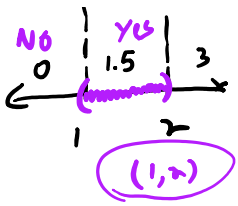
$\frac{x-2x+2}{x-1} > 0$

$\frac{-x+2}{x-1} > 0$ (0 at $-x+2$, 0 at $x-1$)

C.V.'s: 2, 1

TV = 0: $\frac{+}{-1} > 0$ No

TV = 1.5: $\frac{+}{+} > 0$ Yep!



$\frac{TV - 1.5}{+}$

$\frac{}{+} > 0$ Yep!

TV = 3

NO!