Logarithmic Functions

Goals:

1. To evaluate log functions
2. To graph log functions
3. To convert between forms
4. To use in apps

$2^3 = 8$

3 is $\boxed{\text{exponent}}$ on base 2 that produces 8.
$\boxed{\text{logarithm}}$ "="

$$\underbrace{3 = \boxed{\log_2(8)}}_{\text{logarithmic form}} \overset{\text{equivalent}}{\Longleftrightarrow} \underbrace{2^3 = 8}_{\text{exponential form}}$$

Def: $\overset{\text{exp form}}{\boxed{a^y = x}}$ is equivalent to $\boxed{y = \log_a(x)}$ (logarithmic form)

where $a, x > 0$, $a \neq 1$.

Notes: ① $f(x) = \log_a x$ is a function.

② $\boxed{y = \log_a(x)}$ and $\boxed{y = a^x}$ are inverses.

$\boxed{a^y = x}$

(ex) convert from exponential to log form or vice-versa

a) $\boxed{3^4 = 81}$

$4 = \log_3(81)$

b) $\boxed{u^v = w}$

$$\boxed{v = \underbrace{\log}_{\text{exp}}{}_u(w)}$$

c) $v^{x+y} = u$

$x + y = \log_v(u)$

d) $v = \log_8(w)$

$8^v = w$

e) $\log_{10}(x+b) = c$

$10^c = x + b$

(ex) Graph $\boxed{y = \log_2 x}$ (inverse of $\boxed{y = 2^x}$)

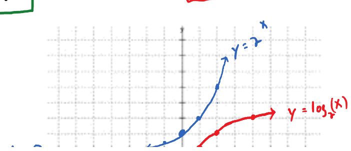
$2^y = x \rightarrow x = 2^y$

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0

$x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$x = 2^{-1} = \frac{1}{2}$

$x = 2^0 = 1$



x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
$\frac{1}{8}$	-3

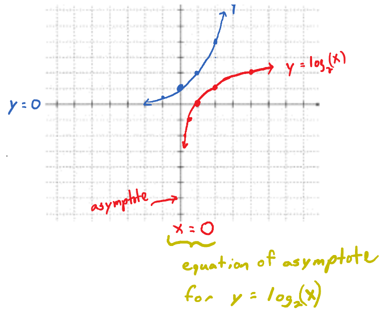
$x = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$x = 2^{-1} = \frac{1}{2}$

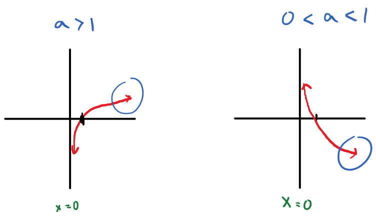
$x = 2^0 = 1$

$x = 2^1 = 2$

$x = 2^2 = 4$



Notes on $f(x) = \log_a x$



1. Asymptote: $x = 0$ (y-axis)
2. Domain: $(0, \infty)$
3. Range: $(-\infty, \infty)$

Properties of logs and exponents

$\boxed{a^1 = a}$ ① $\log_a a = 1$

$a^0 = 1$ ② $\log_a (1) = 0$

$a^x = a^x$ ③ $\log_a (a^x) = x$

④ $a^x = a^y$ $\boxed{\text{iff}}$ $x = y$, $a \neq 0, 1$, $a > 0$

$\underbrace{\Leftrightarrow}_{\text{equivalent}}$

ex solve

a) $\log_4 x = 3$

$x = 4^3$

$x = 64$

b) $\boxed{\log_4 8 = 2x+1}$

$4^{2x+1} = 8$

$(2^2)^{2x+1} = 2^3$

$2^{4x+2} = 2^3$

$4x + 2 = 3$

$4x = 1$

$x = \frac{1}{4}$

ex evaluate without a calculator: $\log_9 \frac{1}{27}$

let $\log_9 \frac{1}{27} = x$

$9^x = \frac{1}{27}$

$(3^2)^x = \frac{1}{3^3}$

$3^{2x} = 3^{-3}$

$2x = -3$

$x = -\frac{3}{2}$

Def: ① $\log x$ means $\log_{10} x$ and is called the common logarithm

② $\ln x = \log_e x$ is called the natural logarithm, where $e \approx 2.718$