

Circles and Ellipses

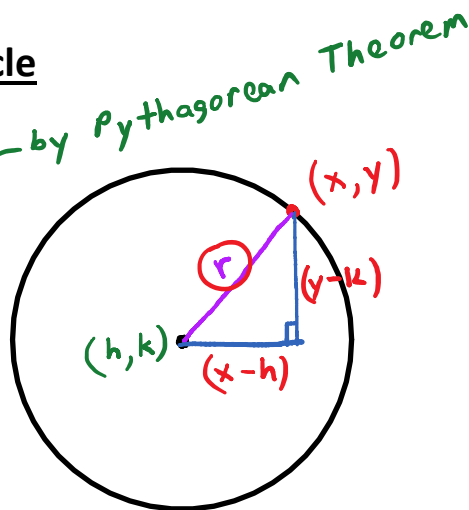
Goals:

1. To find the equation of a circle given its center and radius.
2. To find the center and radius of a circle given its equation.
3. To graph an ellipse.

The Standard Equation of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$C(h, k)$, radius = r



(ex) Find the equation of a circle with the given center and radius.

a) $C(0, 0)$, $r = 1$ (unit circle)

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = 1^2$$

$$x^2 + y^2 = 1$$

b) $C(2, -3)$, $r = 5$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-(-3))^2 = 5^2$$

$$(x-2)^2 + (y+3)^2 = 25$$

$$(x-2)^2 + (y+3)^2 = 25$$

c) C(-4, 5) r = 2√3

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+4)^2 + (y-5)^2 = (2\sqrt{3})^2$$

$$= 2^2 (\sqrt{3})^2$$

$$= 4 \cdot 3$$

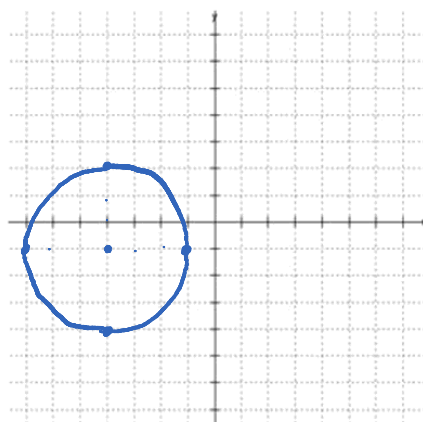
$$(x+4)^2 + (y-5)^2 = 12$$

ex) Find the center and radius and graph the circle with the given equation:

a) $(x+4)^2 + (y-1)^2 = 9$

$$(x-h)^2 + (y-k)^2 = r^2$$

C(-4, 1), r=3



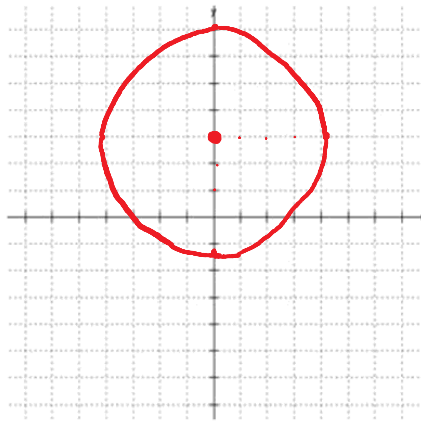
b) $x^2 + (y-3)^2 = 17$

$$(x-h)^2 + (y-k)^2 = r^2$$



$$(x-h)^2 + (y-k)^2 = r^2$$

$$C(0, 3), \quad r = \sqrt{17}$$
$$r = 4.1$$



$$C(0, 0), \quad r = \text{radius}$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

If I replaced r with a bigger number, the graph would be stretched horizontally, creating an oval shape (ellipse)

The Standard Equation of an Ellipse

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}, \quad \text{with } C(0, 0)$$

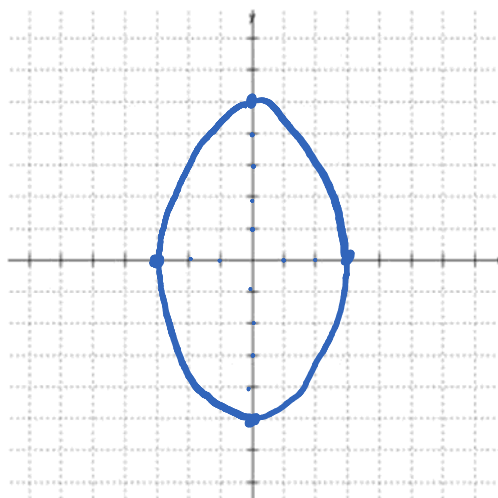
↳ This single equation works as long as we don't insist $a > b$.

ⓧ Graph the ellipse

$$a) \quad \frac{x^2}{9} + \frac{y^2}{25} = 1$$

left/right $\sqrt{9} = 3$

up/down $\sqrt{25} = 5$



$$b) \quad 4x^2 + 9y^2 = 36$$

write
in std.
form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

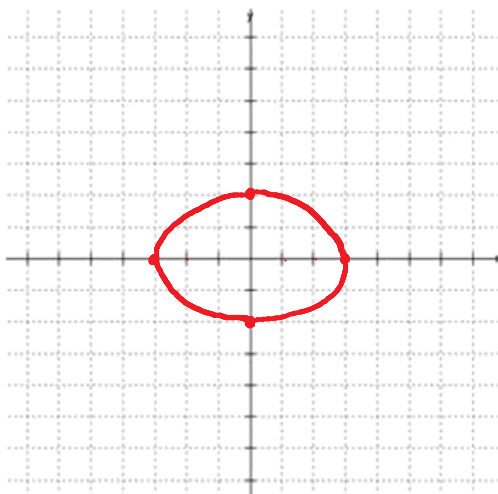
$$\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$C(0,0)$

left/right $\sqrt{9} = 3$

up/down $\sqrt{4} = 2$

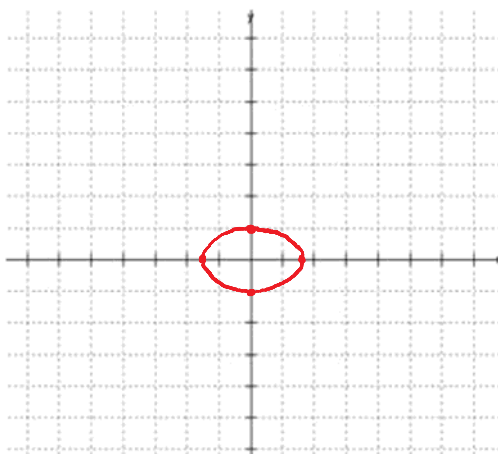


$$c) \quad 6x^2 + 16y^2 = 16$$

$$\frac{6x^2}{16} + \frac{16y^2}{16} = \frac{16}{16}$$

$$\frac{6x^2}{16} + y^2 = 1$$

$$\frac{3x^2}{8} + y^2 = 1$$



$$\frac{6x^2}{16} + \frac{16y^2}{16} = \frac{16}{16}$$

$$\frac{6x^2}{16} + y^2 = 1$$

$$\frac{3x^2}{8} + y^2 = 1$$

$$\frac{x^2}{\frac{8}{3}} + \frac{y^2}{1} = 1$$

left/right $\sqrt{\frac{8}{3}} \approx 1.6$, up/down = $\sqrt{1} = 1$

