Properties of Logarithms

Goals:

1. To apply the product, quotient, and power rules.
2. To apply the Change of Base formula.
3. To graph the natural exponential and natural logarithm functions.

In this section, $M, N, a > 0$ and $a \neq 1$.

Product Rule

$$\log_a(MN) = \underbrace{\log_a M}_{u} + \underbrace{\log_a N}_{v}$$

Proof: Let $\underbrace{u = \log_a M}$ and $\underbrace{v = \log_a N}$

convert to exp. form

$a^u = M$ $\quad$ $a^v = N$

$a^u a^v = MN$

$\underbrace{a^{u+v} = MN}_{\text{log form}}$

$\log_a MN = u + v$

$\log_a MN = \log_a M + \log_a N$

Done.

(ex) Express as a sum of logarithms:

$$\log(5 \cdot 8)$$

$$\underbrace{\log(5 \cdot 8)}_{1.602} = \underbrace{\log(5) + \log(8)}_{1.602}$$

(ex) Write as a single log: $\boxed{\log_4 u + \log_4 v}$

$$\boxed{\log_a(MN) = \log_a M + \log_a N}$$

$$\log_4 u + \log_4 v = \log_4(u \cdot v)$$

$$= \log_4 uv$$

Quotient Rule

$$\boxed{\log_a \frac{M}{N} = \log_a M - \log_a N}$$

$$\log_a M - \log_a N \neq \frac{\log_a M}{\log_a N}$$

ex Write as a single logarithm:

$\log_b 22 - \log_b 3$

$$\log_a \frac{M}{N} = \boxed{\log_a M - \log_a N}$$

$$\log_b 23 - \log_b 3 = \log_b \left(\frac{22}{3}\right)$$

Power Rule

$$\log_b (M^c) = c \log_b M$$

ex $\underbrace{\log_b M^2} = 2 \log_b M$

$$\log_b (M \cdot M) = 1 \log_b M + 1 \log_b M$$

ex Write as a product: $\log_b x^{10}$

$$= 10 \log_b x$$

ex Write as a single logarithm:

$$20 \log_a x$$

$$= \log_a x^{20}$$

Base Conversion

$$\log_b m = \frac{\log_a m}{\log_a b}$$

ex Estimate $\log_3 5$

$$= \frac{\log(5)}{\log(3)} \approx 1.465$$

$$\neq$$

$$= \frac{\ln(5)}{\ln(3)} \approx 1.465$$

Definitions

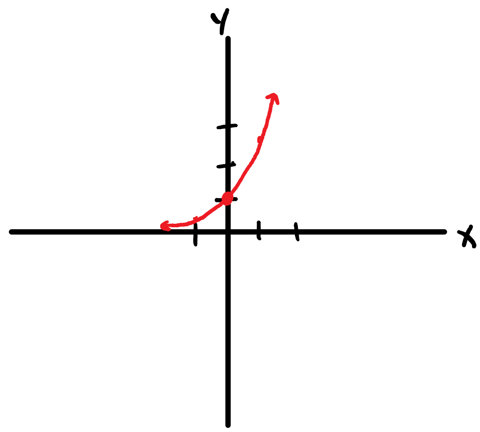
→ ① $\log x = \log_{10} x$ (Common Logarithm)

→ ② $\ln x = \log_e x$ (Natural Logarithm)

where $e \approx 2.718$

ex Graph $y = e^{x}$ (Natural Exponential)

x	$y = e^x$
-1	≈ 0.4
0	1
1	≈ 2.7
2	≈ 7.4



$y = \ln x = \log_e x$ Natural Exponential Function

ex Graph $\frac{y}{2} = \frac{2 \cdot \ln x}{2}$

$\frac{y}{2} = \ln x$

$\frac{y}{2} = \log_e x$

$e^{\frac{y}{2}} = x$

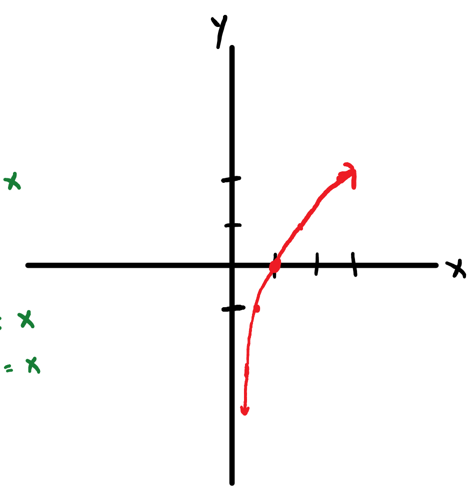
$e^{-\frac{1}{2}} = x$

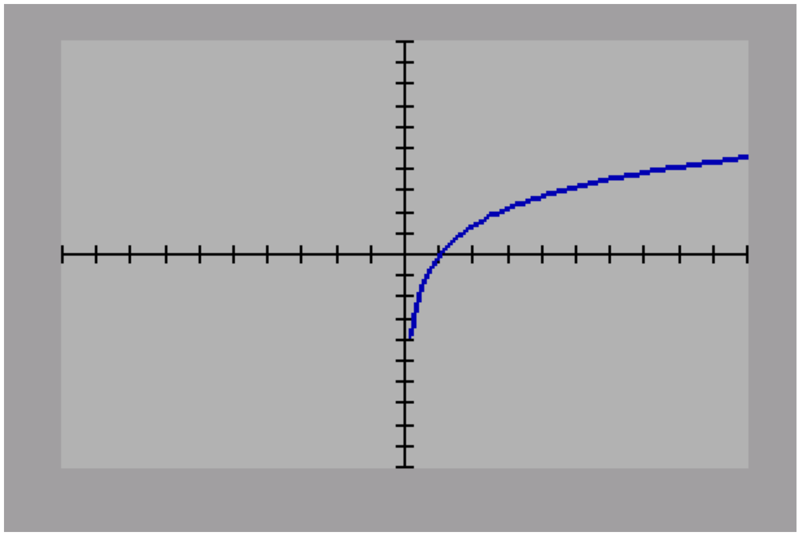
x	y
0.6	-1
1	0
1.6	1
2.7	2

$e^{-\frac{1}{2}} = x$

$e^{\frac{1}{2}} = x$

$e^{1} = x$





5/9/2016 1:43 PM - Screen Clipping